**Primitive network**

A power network is essentially an interconnection of several two-terminal components such as generators, transformers, transmission lines, motors and loads. Each element has an impedance. The voltage across the element is called element voltage and the current flowing through the element is called the element current. A set of components when they are connected form a Primitive network.

A representation of a power system and the corresponding oriented graph are shown in Fig. 2.1.

![Fig. 2.1 A power system and corresponding oriented graph](image-url)
Connectivity various elements to form the network can be shown by the bus incidence matrix $A$. For above system, this matrix is obtained as

$$A = \begin{bmatrix}
-1 & 1
-1 & 1 & -1 & 1
-1 & -1
-1 & 1 & -1
\end{bmatrix}$$

(2.1)
Element voltages are referred as $v_1, v_2, v_3, v_4, v_5, v_6$ and $v_7$. Element currents are referred as $i_1, i_2, i_3, i_4, i_5, i_6$ and $i_7$. In power system network, bus voltages and bus currents are of more useful. For the above network, the bus voltages are $V_1, V_2, V_3$ and $V_4$. The bus voltages are always measured with respect to the ground bus. The bus currents are designated as $I_1, I_2, I_3$, and $I_4$. The element voltages are related to bus voltages as:

$$
\begin{align*}
    v_1 &= -V_1 \\
    v_2 &= -V_2 \\
    v_3 &= -V_4 \\
    v_4 &= V_4 - V_3 \\
    v_5 &= V_2 - V_3 \\
    v_6 &= V_1 - V_2 \\
    v_7 &= V_2 - V_4
\end{align*}
$$
Expressing the relation in matrix form

\[
\begin{bmatrix}
    v_1 \\
    v_2 \\
    v_3 \\
    v_4 \\
    v_5 \\
    v_6 \\
    v_7
\end{bmatrix} =
\begin{bmatrix}
    -1 &  &  &  &  &  & \\
    & -1 &  &  &  &  & \\
    &  & -1 & &  &  & \\
    &  &  & -1 &  &  & \\
    1 &  & -1 &  &  & & \\
    1 &  &  &  & -1 & & \\
    1 &  &  &  &  & -1 &
\end{bmatrix}
\begin{bmatrix}
    v_1 \\
    v_2 \\
    v_3 \\
    v_4 \\
\end{bmatrix}
\]

(2.2)

Thus \( v = A^T \ V_{bus} \)  

(2.3)

The element currents are related to bus currents as:

\[
\begin{align*}
I_1 &= -i_1 + i_6 \\
I_2 &= -i_2 + i_5 - i_6 + i_7 \\
I_3 &= -i_4 - i_5 \\
I_4 &= -i_3 + i_4 - i_7
\end{align*}
\]
Expressing the relation in matrix form

$$\begin{align*}
I_1 &= -i_1 + i_6 \\
I_2 &= -i_2 + i_5 - i_6 + i_7 \\
I_3 &= -i_4 - i_5 \\
I_4 &= -i_3 + i_4 - i_7
\end{align*}$$

Thus $I_{bus} = A i$ 

(2.4)
The element voltages and element impedances are related as:

\[
\begin{bmatrix}
    v_1 \\
    v_2 \\
    v_3 \\
    v_4 \\
    v_5 \\
    v_6 \\
    v_7
\end{bmatrix} =
\begin{bmatrix}
    z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & z_{17} \\
    z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} & z_{27} \\
    z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} & z_{37} \\
    z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} & z_{47} \\
    z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{56} & z_{57} \\
    z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} & z_{67} \\
    z_{71} & z_{72} & z_{73} & z_{74} & z_{75} & z_{76} & z_{77}
\end{bmatrix} \begin{bmatrix}
    i_1 \\
    i_2 \\
    i_3 \\
    i_4 \\
    i_5 \\
    i_6 \\
    i_7
\end{bmatrix}
\]

(2.5)

Here \(z_{ii}\) is the self impedance of element \(i\) and \(z_{ij}\) is the mutual impedance between elements \(i\) and \(j\). In matrix notation the above can be written as

\[
v = z \ i
\]

(2.6)

Here \(z\) is known as primitive impedance matrix. The inverse form of above is

\[
i = y \ v
\]

(2.7)

In the above \(y\) is called as primitive admittance matrix. Matrices \(z\) and \(y\) are inverses of each other.
\( v = z \ i \) \hspace{1cm} (2.6)

\( i = y \ v \) \hspace{1cm} (2.7)

Similar to the above two relations, in terms of bus frame

\[ V_{bus} = Z_{bus} \ I_{bus} \] \hspace{1cm} (2.8)

Here \( V_{bus} \) is the bus voltage vector, \( I_{bus} \) is the bus current vector and \( Z_{bus} \) is the bus impedance matrix. The inverse form of above is

\[ I_{bus} = Y_{bus} \ V_{bus} \] \hspace{1cm} (2.9)

Here \( Y_{bus} \) is known as bus impedance matrix. Matrices \( Z_{bus} \) and \( Y_{bus} \) are inverses of each other.
Derivation of bus admittance matrix

It was shown that

\[ v = A^T V_{bus} \] (2.3)

\[ I_{bus} = A i \] (2.4)

\[ i = y v \] (2.7)

\[ I_{bus} = Y_{bus} V_{bus} \] (2.9)

Substituting eq. (2.7) in eq. (2.4)

\[ I_{bus} = A y v \] (2.10)

Substituting eq. (2.3) in the above

\[ I_{bus} = A y A^T V_{bus} \] (2.11)

Comparing eqs. (2.9) and (2.11)

\[ Y_{bus} = A y A^T \] (2.12)

This is a very general formula for bus admittance matrix and admits mutual coupling between elements.
In power system problems mutual couplings will have negligible effect and often omitted. In that case the primitive impedance matrix $z$ and the primitive admittance matrix $y$ are diagonal and $Y_{bus}$ can be obtained by inspection. This is illustrated through the seven-elements network considered earlier. When mutual couplings are neglected

$$\begin{bmatrix} y_{11} & & & & & & \\ & y_{22} & & & & & \\ & & y_{33} & & & & \\ & & & y_{44} & & & \\ & & & & y_{55} & & \\ & & & & & y_{66} & \\ & & & & & & y_{77} \end{bmatrix}$$

(2.13)

$$Y_{bus} = A \ y \ A^T$$

$$\begin{bmatrix} y_{11} & & & & & & \\ & y_{22} & & & & & \\ & & y_{33} & & & & \\ & & & y_{44} & & & \\ & & & & y_{55} & & \\ & & & & & y_{66} & \\ & & & & & & y_{77} \end{bmatrix}$$

$$\begin{array}{cccc}
-1 & & & \\
& -1 & & \\
& & -1 & \\
& & & 1 & 1 \\
& & & 1 & -1 \\
& & & & 1 & -1 \\
& & & & & 1 & -1 \\
\end{array}$$
\[ Y_{bus} = \begin{bmatrix}
1 & 2 & 3 & 4 \\
1 & y_{11} + y_{66} & -y_{66} & 0 & 0 \\
2 & -y_{66} & y_{22} + y_{55} + y_{66} + y_{77} & -y_{55} & -y_{77} \\
3 & 0 & -y_{55} & y_{44} + y_{55} & -y_{44} \\
4 & 0 & -y_{77} & -y_{44} & y_{33} + y_{44} + y_{77}
\end{bmatrix} \]
The rules to form the elements of $Y_{bus}$ are:

- The diagonal element $Y_{ii}$ equals the sum of the admittances directly connected to bus $i$.
- The off-diagonal element $Y_{ij}$ equals the negative of the admittance connected between buses $i$ and $j$. If there is no element between buses $i$ and $j$, then $Y_{ij}$ equals to zero.
Bus admittance matrix can be constructed by adding the elements one by one. Separating the entries corresponding to the element 5 that is connected between buses 2 and 3 the above $Y_{bus}$ can be written as

\[
Y_{bus} = \begin{bmatrix}
  y_{11} + y_{66} & -y_{66} & 0 & 0 \\
  -y_{66} & y_{22} + y_{66} + y_{77} & 0 & -y_{77} \\
  0 & 0 & y_{44} & -y_{44} \\
  0 & -y_{77} & -y_{44} & y_{33} + y_{44} + y_{77}
\end{bmatrix}
\]

It can be inferred that the effect of adding element 5 between buses 2 and 3 is to add admittance $y_{55}$ to elements $Y_{bus}(2,2)$ and $Y_{bus}(3,3)$ and add $-y_{55}$ to elements $Y_{bus}(2,3)$ and $Y_{bus}(3,2)$. To construct the bus admittance matrix $Y_{bus}$, initially all the elements are set to zero; then network elements are added one by one, each time four elements of $Y_{bus}$ are modified.
Example 2.1

Consider the power network shown in Fig. 2.2. The ground bus is marked as 0. Grounding impedances at buses 1, 2, and 4 are j0.6 Ω, j0.4 Ω and j0.5 Ω respectively. Impedances of the elements 3-4, 2-3, 1-2 and 2-4 are j0.25 Ω, j0.2 Ω, j0.2 Ω and j0.5 Ω. The mutual impedance between elements 2-3 and 2-4 is j0.1 Ω. Obtain the bus admittance matrix of the power network.

Fig. 2.2 Power network – Example 2.1
Solution

The oriented graph of the network, with impedances marked is shown in Fig. 2.3.

![Graph of the network with impedances](image)

Fig. 2.3 Data for Example 2.1

Primitive impedance matrix is:

\[
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
1 & 0.6 & & & & & \\
2 & & 0.4 & & & & \\
3 & & & 0.5 & & & \\
4 & & & & 0.25 & & \\
5 & & & & & 0.2 & 0.1 \\
6 & & & & & & 0.2 \\
7 & & & & & & 0.1 & 0.5 \\
\end{array}
\]
Inverting this

\[
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
1 & 1.6667 & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
2 & \text{ } & 2.5 & \text{ } & \text{ } & \text{ } & \text{ } \\
3 & \text{ } & \text{ } & 2.0 & \text{ } & \text{ } & \text{ } \\
4 & \text{ } & \text{ } & \text{ } & 4.0 & \text{ } & \text{ } \\
5 & \text{ } & \text{ } & \text{ } & \text{ } & 5.5556 & -1.1111 \\
6 & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & 5.0 \\
7 & \text{ } & \text{ } & \text{ } & \text{ } & -1.1111 & 2.2222 \\
\end{array}
\]

Bus incidence matrix A is:

\[
A =
\begin{array}{ccccccc}
-1 & \text{ } & \text{ } & \text{ } & 1 & \text{ } & \text{ } \\
-1 & 1 & -1 & 1 & \text{ } & \text{ } & \text{ } \\
-1 & -1 & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
-1 & 1 & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
\end{array}
\]
Bus admittance matrix $Y_{bus} = A y A^T$

\[
Y_{bus} = -j A
\]

\[
Y_{bus} = \begin{bmatrix}
1.6667 & -1 \\
2.5 & -1 \\
2.0 & -1 \\
4.0 & -1 \\
5.5556 & -1 \\
5.0 & -1 \\
-1.1111 & -1 \\
2.2222 & -1 \\
\end{bmatrix}
\]

\[
Y_{bus} = -j
\]

\[
Y_{bus} = \begin{bmatrix}
1 & 1 & -1 & 1 \\
-1 & 1 & -1 & 1 \\
-1 & -1 & -1 & 1 \\
-1 & 1 & -1 & -1 \\
\end{bmatrix}
\]

\[
Y_{bus} = \begin{bmatrix}
1.6667 & -1.6667 & 0 & 0 \\
-2.5 & -2.5 & 0 & 0 \\
-2.0 & -2.0 & 0 & 0 \\
-4.0 & -4.0 & 0 & 0 \\
4.4444 & 4.4444 & 0 & 0 \\
-5.5556 & -5.5556 & 0 & 0 \\
1.1111 & 1.1111 & 0 & 0 \\
-2.2222 & -2.2222 & 0 & 0 \\
\end{bmatrix}
\]

\[
Y_{bus} = \begin{bmatrix}
1 & 2 & 3 & 4 \\
1 & -j6.6667 & j5.0 & 0 & 0 \\
2 & j5.0 & -j13.0556 & j4.4444 & j1.1111 \\
3 & 0 & j4.4444 & -j9.5556 & j5.1111 \\
4 & 0 & j1.1111 & j5.1111 & -j8.2222 \\
\end{bmatrix}
\]
Example 2.2

Neglect the mutual impedance and obtain $Y_{bus}$ for the power network described in example 2.1.

Solution

Admittances of elements 1 to 7 are

- $-j1.6667$, - $j2.5$, - $j2.0$, - $j4.0$, - $j5.0$, - $j5.0$ and – $j2.0$. They are marked blow.

$$Y_{bus} = \begin{bmatrix}
- j6.6667 & j5.0 & 0 & 0 \\
- j5.0 & - j14.5 & j5.0 & j2.0 \\
0 & j5.0 & - j9.0 & j4.0 \\
0 & j2.0 & j4.0 & - j8.0
\end{bmatrix}$$
Example 2.3

Repeat previous example by adding elements one by one.

Solution

Initially all the elements of $Y_{bus}$ are set to zeros.

Add element 1: It is between 0-1 with admittance – $j1.6667$

\[
Y_{bus} = \begin{bmatrix}
- j1.6667 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Add element 2: It is between 0-2 with admittance – $j2.5$

\[
Y_{bus} = \begin{bmatrix}
- j1.6667 & 0 & 0 & 0 \\
0 & - j2.5 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
Add element 3: It is between 0-4 with admittance – j2

\[
Y_{\text{bus}} = \begin{bmatrix}
1 & 2 & 3 & 4 \\
1 & -j1.6667 & 0 & 0 & 0 \\
2 & 0 & -j2.5 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & -j2.0 \\
\end{bmatrix}
\]

Add element 4: It is between 3-4 with admittance – j4

\[
Y_{\text{bus}} = \begin{bmatrix}
1 & 2 & 3 & 4 \\
1 & -j1.6667 & 0 & 0 & 0 \\
2 & 0 & -j2.5 & 0 & 0 \\
3 & 0 & 0 & -j4.0 & j4.0 \\
4 & 0 & 0 & j4.0 & -j6.0 \\
\end{bmatrix}
\]
Add element 5: It is between 2-3 with admittance – j5

\[
Y_{bus} = \begin{bmatrix}
1 & 2 & 3 & 4 \\
1 & -j1.6667 & 0 & 0 & 0 \\
2 & 0 & -j7.5 & j5.0 & 0 \\
3 & 0 & j5.0 & -j9.0 & j4.0 \\
4 & 0 & 0 & j4.0 & -j6.0 \\
\end{bmatrix}
\]

Add element 6: It is between 1-2 with admittance – j5

\[
Y_{bus} = \begin{bmatrix}
1 & 2 & 3 & 4 \\
1 & -j6.6667 & j5.0 & 0 & 0 \\
2 & j5.0 & -j12.5 & j5.0 & 0 \\
3 & 0 & j5.0 & -j9.0 & j4.0 \\
4 & 0 & 0 & j4.0 & -j6.0 \\
\end{bmatrix}
\]

Add element 7: It is between 2-4 with admittance – j2. Final bus admittance matrix

\[
Y_{bus} = \begin{bmatrix}
1 & 2 & 3 & 4 \\
1 & -j6.6667 & j5.0 & 0 & 0 \\
2 & j5.0 & -j14.5 & j5.0 & j2.0 \\
3 & 0 & j5.0 & -j9.0 & j4.0 \\
4 & 0 & j2.0 & j4.0 & -j8.0 \\
\end{bmatrix}
\]
NETWORK REDUCTION

For a four node network, the performance equations in bus frame using the admittance parameter can be written as:

\[
\begin{bmatrix}
    I_1 \\
    I_2 \\
    I_3 \\
    I_4 \\
\end{bmatrix} =
\begin{bmatrix}
    Y_{11} & Y_{12} & Y_{13} & Y_{14} \\
    Y_{21} & Y_{22} & Y_{23} & Y_{24} \\
    Y_{31} & Y_{32} & Y_{33} & Y_{34} \\
    Y_{41} & Y_{42} & Y_{43} & Y_{44} \\
\end{bmatrix}
\begin{bmatrix}
    V_1 \\
    V_2 \\
    V_3 \\
    V_4 \\
\end{bmatrix}
\]

Suppose current \( I_4 = 0 \), the node 4 can be eliminated and the network equations can be written as:

\[
\begin{bmatrix}
    I_1 \\
    I_2 \\
    I_3 \\
\end{bmatrix} =
\begin{bmatrix}
    Y'_{11} & Y'_{12} & Y'_{13} \\
    Y'_{21} & Y'_{22} & Y'_{23} \\
    Y'_{31} & Y'_{32} & Y'_{33} \\
\end{bmatrix}
\begin{bmatrix}
    V_1 \\
    V_2 \\
    V_3 \\
\end{bmatrix}
\]

Consider the first set of equations. The equation corresponding to node 4 is

\[ Y_{41} V_1 + Y_{42} V_2 + Y_{43} V_3 + Y_{44} V_4 = 0 \]

Thus,

\[ V_4 = -\frac{Y_{41}}{Y_{44}} V_1 - \frac{Y_{42}}{Y_{44}} V_2 - \frac{Y_{43}}{Y_{44}} V_3 \]
Thus, \[ V_4 = -\frac{Y_{41}}{Y_{44}} V_1 - \frac{Y_{42}}{Y_{44}} V_2 - \frac{Y_{43}}{Y_{44}} V_3 \]

Substituting the above in the equation of node 1

\[ I_1 = Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3 + Y_{14} \left( -\frac{Y_{41}}{Y_{44}} V_1 - \frac{Y_{42}}{Y_{44}} V_2 - \frac{Y_{43}}{Y_{44}} V_3 \right) \]

\[ \left( Y_{11} - \frac{Y_{14} Y_{41}}{Y_{44}} \right) V_1 + \left( Y_{12} - \frac{Y_{14} Y_{42}}{Y_{44}} \right) V_2 + \left( Y_{13} - \frac{Y_{14} Y_{43}}{Y_{44}} \right) V_3 \]

Comparing the above with \( I_1 = Y_{11}' V_1 + Y_{12}' V_2 + Y_{13}' V_3 \)

\[ Y_{11}' = Y_{11} - \frac{Y_{14} Y_{41}}{Y_{44}} \]
\[ Y_{12}' = Y_{12} - \frac{Y_{14} Y_{42}}{Y_{44}} \]
\[ Y_{13}' = Y_{13} - \frac{Y_{14} Y_{43}}{Y_{44}} \]
Thus in general, when node $k$ is eliminated, the modified elements can be calculated as

\[
Y_{11}' = Y_{11} - \frac{Y_{14}Y_{41}}{Y_{44}}
\]

\[
Y_{12}' = Y_{12} - \frac{Y_{14}Y_{42}}{Y_{44}}
\]

\[
Y_{13}' = Y_{13} - \frac{Y_{14}Y_{43}}{Y_{44}}
\]

Thus in general, when node $k$ is eliminated, the modified elements can be calculated as

\[
Y_{ij}' = Y_{ij} - \frac{Y_{ik}Y_{kj}}{Y_{kk}}
\]

where $i = 1, 2, \ldots, N$  $i \neq k$  and  $j = 1, 2, \ldots, N$  $j \neq k$
Example

Solve the equations

\[
\begin{bmatrix}
0.625 & -0.5 & 0 & 0 \\
-0.5 & 1.0833 & -0.25 & -0.3333 \\
0 & -0.25 & 0.75 & -0.5 \\
0 & -0.3333 & -0.5 & 0.8333
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
2 \\
0 \\
4
\end{bmatrix}
\]

for the node voltages using network reduction.

Solution

Eliminating node 1, we get

\[
\begin{bmatrix}
0.6833 & -0.25 & -0.3333 \\
-0.25 & 0.75 & -0.5 \\
-0.3333 & -0.5 & 0.8333
\end{bmatrix}
\begin{bmatrix}
V_2 \\
V_3 \\
V_4
\end{bmatrix}
= 
\begin{bmatrix}
2 \\
0 \\
4
\end{bmatrix}
\]
Eliminating node 1, we get

\[
\begin{bmatrix}
0.6833 & -0.25 & -0.3333 \\
-0.25 & 0.75 & -0.5 \\
-0.3333 & -0.5 & 0.8333
\end{bmatrix}
\begin{bmatrix}
V_2 \\
V_3 \\
V_4
\end{bmatrix}
= 
\begin{bmatrix}
2 \\
0 \\
4
\end{bmatrix}
\]

Eliminating node number 3, we get

\[
\begin{bmatrix}
0.6 & -0.5 \\
-0.5 & 0.5
\end{bmatrix}
\begin{bmatrix}
V_2 \\
V_4
\end{bmatrix}
= 
\begin{bmatrix}
2 \\
4
\end{bmatrix}
\]

On solving the above, \( V_2 = 60 \) and \( V_4 = 68 \)
\[
\begin{bmatrix}
0.625 & -0.5 & 0 & 0 \\
-0.5 & 1.0833 & -0.25 & -0.3333 \\
0 & -0.25 & 0.75 & -0.5 \\
0 & -0.3333 & -0.5 & 0.8333 \\
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
2 \\
0 \\
4 \\
\end{bmatrix}
\]

The node 1 equation of first set gives

\[0.625 V_1 - 0.5 V_2 = 0\quad \text{Thus} \quad V_1 = \frac{0.5 \times 60}{0.625} = 48\]

The node 3 equation of first set gives

\[-0.25 V_2 + 0.75 V_3 - 0.5 V_4 = 0\quad \text{Thus} \quad V_3 = \frac{0.25 \times 60 + 0.5 \times 68}{0.75} = 65.3333\]

Thus

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
\end{bmatrix}
= 
\begin{bmatrix}
48 \\
60 \\
65.3333 \\
68 \\
\end{bmatrix}
\]
Formulation of Power Flow problem

Power flow analysis is the most fundamental study to be performed in a power system both during the Planning and Operational phases. It constitutes the major portion of electric utility. The study is concerned with the normal steady state operation of power system and involves the determination of bus voltages and power flows for a given network configuration and loading condition.

The results of power flow analysis help to know

1. the present status of the power system, required for continuous monitoring.
2. alternative plans for system expansion to meet the ever increasing demand.

The mathematical formulation of the power flow problem results in a system of non-linear algebraic equations and hence calls for an iterative technique for obtaining the solution. Gauss-Seidel method and Newton Raphson (N.R.) method are commonly used to get the power flow solution.
With reasonable assumptions and approximations, a power system may be modeled as shown in Fig. 2.4 for purpose of steady state analysis.
The model consists of a network in which a number of buses are interconnected by means of lines which may either transmission lines or power transformers. The generators and loads are simply characterized by the complex powers flowing into and out of buses respectively. Each transmission line is characterized by a lumped impedance and a line charging capacitance. Static capacitors or reactors may be located at certain buses either to boost or buck the load-bus voltages at times of need.

Thus the Power Flow problem may be stated as follows:

Given the network configuration and the loads at various buses, determine a schedule of generation so that the bus voltages and hence line flows remain within security limits.
A more specific statement of the problem will be made subsequently after taking into consideration the following three observations.

1. For a given load, we can arbitrarily select the schedules of all the generating buses, except one, to lie within the allowable limits of the generation. The generation at one of the buses, called as the slack bus, cannot be specified beforehand since the total generation should be equal to the total demand plus the transmission losses, which is not known unless all the bus voltages are determined.

2. Once the complex voltages at all the buses are known, all other quantities of interest such as line flows, transmission losses and generation at the buses can easily be determined. Hence the foremost aim of the power flow problem is to solve for the bus voltages.
It will be convenient to use the Bus Power Specification which is defined as the difference between the specified generation and load at a bus. Thus for the \( k^{th} \) bus, the bus power specification \( S_k \) is given by

\[
S_k = P_l_k + jQ_l_k \\
= (P_{G_k} + jQ_{G_k}) - (P_{D_k} + jQ_{D_k}) \\
= (P_{G_k} - P_{D_k}) + j(Q_{G_k} - Q_{D_k})
\]

(2.14)

In view of the above three observations Power Flow Problem may be defined as that of determining the complex voltages at all the buses, given the network configuration and the bus power specifications at all the buses except the slack bus.
Classification of buses

There are four quantities associated with each bus. They are $P_I$, $Q_I$, $V_I$, and $\delta$.

Here $P_I$ is the real power injected into the bus.

$Q_I$ is the reactive power injected into the bus.

$V_I$ is the magnitude of the bus voltage.

$\delta$ is the phase angle of the bus voltage.

Any two of these four may be treated as independent variables (i.e. specified) while the other two may be computed by solving the power flow equations. Depending on which of the two variables are specified, buses are classified into three types. Three types of bus classification based on practical requirements are shown below.

![Three types of buses](image)

Fig. 2.5 Three types of buses
Slack bus

In a power system with N buses, power flow problem is primarily concerned with determining the 2N bus voltage variables, namely the voltage magnitude and phase angles. These can be obtained by solving the 2N power flow equations provided there are 2N power specifications. However as discussed earlier the real and reactive power injection at the SLACK BUS cannot be specified beforehand.

This leaves us with no other alternative but to specify two variables $|V_s|$ and $\delta_s$ arbitrarily for the slack bus so that 2(N-1) variables can be solved from 2(N-1) known power specifications.

Incidentally, the specification of $|V_s|$ helps us to fix the voltage level of the system and the specification of $\delta_s$ serves as the phase angle reference for the system.

Thus for the slack bus, both $|V|$ and $\delta$ are specified and PI and QI are to be determined. PI and QI can be computed at the end, when all the $|V|$s and $\delta$s are solved for.
**Generator bus**

In a generator bus, it is customary to maintain the bus voltage magnitude at a desired level which can be achieved in practice by proper reactive power injection. Such buses are termed as the Voltage Controlled Buses or P–V buses. In these buses, $P_I$ and $|V|$ are specified and $Q_I$ and $\delta$ are to be solved for.

**Load bus**

The buses where there is no controllable generation are called as Load Buses or P–Q buses. At the load buses, both $P_I$ and $Q_I$ are specified and $|V|$ and $\delta$ are to be solved for.
Iterative solution for solving power flow model

The power flow model will comprise of a set of simultaneous non-linear algebraic equations. The following two methods are used to solve the power flow model.

1. Gauss-Seidel method

2. Newton Raphson method

Gauss-Seidel method

Gauss-Seidel method is used to solve a set of algebraic equations. Consider

\[ a_{11} x_1 + a_{12} x_2 + \cdots + a_{1N} x_N = y_1 \]
\[ a_{21} x_1 + a_{22} x_2 + \cdots + a_{2N} x_N = y_2 \]
\[ \vdots \quad \vdots \quad \vdots \quad \vdots \]
\[ a_{N1} x_1 + a_{N2} x_2 + \cdots + a_{NN} x_N = y_N \]
Specifically

\[ a_{k1}x_1 + a_{k2}x_2 + \cdots + a_{kk}x_k + \cdots + a_{kN}x_N = y_k \]

Thus

\[ a_{kk}x_k = y_k - \sum_{m=1}^{N} a_{km}x_m \]

This gives

\[ x_k = \frac{1}{a_{kk}} \left[ y_k - \sum_{m=1}^{N} a_{km}x_m \right] \]

\[ k = 1,2,\ldots,N \]

In Gauss-Seidel method, initially, values of \( x_1, x_2, \ldots, x_N \) are assumed. Updated values are calculated using the above equation. In any iteration \( h+1 \), up to \( m = k-1 \), values of \( x_m \) calculated in \( h+1 \) iteration are used and for \( m = k+1 \) to \( N \), values of \( x_m \) calculated in \( h \) iteration are used. Thus

\[ x_k^{h+1} = \frac{1}{a_{kk}} \left[ y_k - \sum_{m=1}^{k-1} a_{km}x_m^{h+1} - \sum_{m=k+1}^{N} a_{km}x_m^{h} \right] \]

(2.15)
Gauss-Seidel method for power flow solution

In this method, first an initial estimate of bus voltages is assumed. By substituting this estimate in the given set of equations, a second estimate, better than the first one, is obtained. This process is repeated and better and better estimates of the solution are obtained until the difference between two successive estimates becomes lesser than a prescribed tolerance.

First let us consider a power system without any P-V bus. Later, the modification required to include the P-V busses will be discussed. This means that given the net power injection at all the load bus, it is required to find the bus voltages at all the load busses.

The expression for net power injection being $V_k I_k^*$, the equations to be solved are

$$V_k I_k^* = P_{I_k} + j Q_{I_k} \quad \text{for} \quad k = 1, 2, \ldots, N$$

(2.16)
\[ V_k I_k^* = P I_k + j Q I_k \quad \text{for} \quad k=1,2,\cdots,N \quad (2.16) \]

In the above equations bus currents \( I_k \) are the intermediate variables that are to be eliminated. Taking conjugate of the above equations yields

\[ V_k^* I_k = P I_k - j Q I_k \quad (2.17) \]

Therefore

\[ I_k = \frac{P I_k - j Q I_k}{V_k^*} \quad (2.18) \]

From the network equations

\[
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{bmatrix} =
\begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1N} \\
Y_{21} & Y_{22} & \cdots & Y_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{N1} & Y_{N2} & \cdots & Y_{NN}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{bmatrix} \quad (2.19)
\]
we can write

\[ I_k = Y_{k1} V_1 + Y_{k2} V_2 + \ldots + Y_{kk} V_k + \ldots + Y_{kN} V_N \]

\[ = Y_{kk} V_k + \sum_{m=1}^{N} Y_{km} V_m \]  \hspace{1cm} (2.20)

Combining equations (2.20) and (2.18), we have

\[ Y_{kk} V_k + \sum_{m=1}^{N} Y_{km} V_m = \frac{P_{I_k} - QI_k}{V_k^*} \]

Thus

\[ V_k = \frac{1}{Y_{kk}} \left[ \frac{P_{I_k} - jQI_k}{V_k^*} - \sum_{m=1}^{N} \frac{Y_{km} V_m}{Y_{kk}} \right] \]

\[ k=1,2,\ldots,N \]

\[ k \neq s \]

\[ = \frac{P_{I_k} - jQI_k}{Y_{kk}} \frac{1}{V_k^*} - \sum_{m=1}^{N} \frac{Y_{km}}{Y_{kk}} V_m \]  \hspace{1cm} (2.21)

\[ k=1,2,\ldots,N \]

\[ k \neq s \]
\[ V_k = \frac{P_l_i - jQ_l_i}{Y_{kk}} \cdot \frac{1}{V_k^*} - \sum_{m=1}^{N} \frac{Y_{km}}{Y_{kk}} V_m \]  \hspace{1cm} (2.21)

A significant reduction in computing time for a solution can be achieved by performing as many arithmetic operations as possible before initiating the iterative calculation. Let us define

\[ \frac{P_l_i - jQ_l_i}{Y_{kk}} = A_k \]  \hspace{1cm} (2.22)

and

\[ \frac{Y_{km}}{Y_{kk}} = B_{km} \]  \hspace{1cm} (2.23)

Having defined \( A_k \) and \( B_{km} \), equation (2.21) becomes

\[ V_k = \frac{A_k}{V_k^*} - \sum_{m=1}^{N} \frac{B_{km} V_m}{Y_{kk}} \quad k=1,2,\ldots, N \quad k \neq s \]  \hspace{1cm} (2.24)

When Gauss-Seidel iterative procedure is used, the voltage at the \( k^{th} \) bus during \( h+1^{th} \) iteration, can be computed as

\[ V_k^{h+1} = \frac{A_k}{V_k^h} - \sum_{m=1}^{k-1} B_{km} V_m^{h+1} - \sum_{m=k+1}^{N} B_{km} V_m^h \quad \text{for} \ k=1,2,\ldots, N; \ k \neq s \]  \hspace{1cm} (2.25)
Line flow equations

Knowing the bus voltages, the power in the lines can be computed as shown below.

\[
I_{km} = (V_k - V_m) y_{km} + V_k y'_{km}
\]

(2.26)

Power flow from bus \( k \) to bus \( m \) is

\[
P_{km} + jQ_{km} = V_k I'_{km}
\]

(2.27)

Substituting equation (2.26) in equation (2.27)

\[
P_{km} + jQ_{km} = V_k [(V_k - V_m^*) y_{km} + V_k^* y'_{km}^*]
\]

(2.28)
\[ P_{km} + jQ_{km} = V_k \left[ (V_k^* - V_m^*) y_{km}^* + V_k^* y_{km}^* \right] \] 

(2.28)

Similarly, power flow from bus \( m \) to bus \( k \) is

\[ P_{mk} + jQ_{mk} = V_m \left[ (V_m^* - V_k^*) y_{km}^* + V_m^* y_{km}^* \right] \] 

(2.29)

The line loss in the transmission line \( k-m \) is given by

\[ P_{L_{k-m}} = (P_{km} + jQ_{km}) + (P_{mk} + jQ_{mk}) \] 

(2.30)

Total transmission loss in the system is

\[ P_L = \sum_{i-j} P_{L_{i-j}} \] 

(2.31)
Example 2.4

For a power system, the transmission line impedances and the line charging admittances in p.u. on a 100 MVA base are given in Table 1. The scheduled generations and loads on different buses are given in Table 2. Taking the slack bus voltage as $1.06 + j 0.0$ and using a flat start perform the power flow analysis and obtain the bus voltages, transmission loss and slack bus power.

Table 1 Transmission line data:

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Bus code k - m</th>
<th>Line Impedance $z_{km}$</th>
<th>HLCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 – 2</td>
<td>0.02 + j 0.06</td>
<td>j 0.030</td>
</tr>
<tr>
<td>2</td>
<td>1 – 3</td>
<td>0.08 + j 0.24</td>
<td>j 0.025</td>
</tr>
<tr>
<td>3</td>
<td>2 – 3</td>
<td>0.06 + j 0.18</td>
<td>j 0.020</td>
</tr>
<tr>
<td>4</td>
<td>2 – 4</td>
<td>0.06 + j 0.18</td>
<td>j 0.020</td>
</tr>
<tr>
<td>5</td>
<td>2 – 5</td>
<td>0.04 + j 0.12</td>
<td>j 0.015</td>
</tr>
<tr>
<td>6</td>
<td>3 – 4</td>
<td>0.01 + j 0.03</td>
<td>j 0.010</td>
</tr>
<tr>
<td>7</td>
<td>4 – 5</td>
<td>0.08 + j 0.24</td>
<td>j 0.025</td>
</tr>
</tbody>
</table>
Table 2  Bus data:

<table>
<thead>
<tr>
<th>Bus code k</th>
<th>Generation</th>
<th>Load</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PG_k in MW</td>
<td>QG_k in MVAR</td>
<td>PD_k in MW</td>
</tr>
<tr>
<td>1</td>
<td>---</td>
<td>---</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>45</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>60</td>
</tr>
</tbody>
</table>

Solution

Flat start means all the unknown voltage magnitude are taken as 1.0 p.u. and all unknown voltage phase angles are taken as 0.

Thus initial solution is

\[ V_1 = 1.06 + j0 \]

\[ V_2^{(0)} = V_3^{(0)} = V_4^{(0)} = V_5^{(0)} = 1.0 + j0 \]
STEP 1

For the transmission system, the bus admittance matrix is to be calculated.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Bus code k - m</th>
<th>Line Impedance ( Z_{km} )</th>
<th>Line admittance ( y_{km} )</th>
<th>HLCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 – 2</td>
<td>0.02 + j 0.06</td>
<td>5 - j 15</td>
<td>j 0.030</td>
</tr>
<tr>
<td>2</td>
<td>1 – 3</td>
<td>0.08 + j 0.24</td>
<td>1.25 – j 3.75</td>
<td>j 0.025</td>
</tr>
<tr>
<td>3</td>
<td>2 – 3</td>
<td>0.06 + j 0.18</td>
<td>1.6667 – j 5</td>
<td>j 0.020</td>
</tr>
<tr>
<td>4</td>
<td>2 – 4</td>
<td>0.06 + j 0.18</td>
<td>1.6667 – j 5</td>
<td>j 0.020</td>
</tr>
<tr>
<td>5</td>
<td>2 – 5</td>
<td>0.04 + j 0.12</td>
<td>2.5 – j 7.5</td>
<td>j 0.015</td>
</tr>
<tr>
<td>6</td>
<td>3 – 4</td>
<td>0.01 + j 0.03</td>
<td>10 – j 30</td>
<td>j 0.010</td>
</tr>
<tr>
<td>7</td>
<td>4 – 5</td>
<td>0.08 + j 0.24</td>
<td>1.25 – j 3.75</td>
<td>j 0.025</td>
</tr>
</tbody>
</table>

\[ Y_{22} = (5 – j15) + (1.6667 – j5) + (1.6667 – j5) + (2.5 – j7.5) + j 0.03 + j 0.02 + j 0.02 + j 0.015 \]
\[ = 10.8334 – j 32.415 \]

Similarly: \( Y_{33} = 12.9167 – j 38.695 \); \( Y_{44} = 12.9167 – j 38.695 \); \( Y_{55} = 3.75 – j 11.21 \)

\[
Y = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
1 & -5 + j15 & 10.8334-j32.415 & -1.6667 + j5 & -1.6667+j5 & -2.5 + j7.5 \\
2 & -1.25 + j3.75 & -1.6667 + j5 & 12.9167–j38.695 & -10 + j30 & 0 \\
3 & 0 & -1.6667 + j5 & -10 + j30 & 12.9167-j38.695 & -1.25 + j3.75 \\
4 & 0 & -2.5+j7.5 & 0 & -1.25+j3.75 & 3.75-j11.21 \\
5 & & & & &
\end{bmatrix}
\]
**STEP 2**

Calculation of elements of A vector and B matrix.

\[ A_k = \frac{P_{I_k} - jQ_{I_k}}{Y_{kk}} \quad \text{and} \quad B_{km} = \frac{Y_{km}}{Y_{kk}} \]

\[ P_{I_2} + jQ_{I_2} = \frac{1}{100} (20 + j20) = 0.2 + j0.2 \]

\[ P_{I_2} - jQ_{I_2} = 0.2 - j0.2 \]

\[ A_2 = \frac{P_{I_2} - jQ_{I_2}}{Y_{22}} = \frac{0.2 - j0.2}{10.8334 - j32.415} = 0.0074 + j0.0037 \]

Similarly \( A_3, A_4 \) and \( A_5 \) are calculated.

\[ B_{21} = \frac{Y_{21}}{Y_{22}} = \frac{-5 + j15}{10.8334 - j32.415} = -0.46263 + j0.00036 \]

Other elements of matrix B can be calculated in a similar manner.
Thus

\[
A = \begin{bmatrix}
1 \\
2 & 0.00740 + j 0.00370 \\
3 & -0.00698 - j 0.00930 \\
4 & -0.00427 - j 0.00891 \\
5 & -0.02413 - j 0.04545 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
-0.46263 + j 0.00036 \\
-0.09690 + j 0.00004 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
-0.12920 \\
-0.12920 + j 0.00006 \\
-0.66881 + j 0.00072 \\
-0.33440 + j 0.00033 \\
-0.23131 + j 0.00018 \\
\end{bmatrix}
\]
STEP 3

Iterative computation of bus voltage can be carried out as shown.

New estimate of voltage at bus 2 is calculated as:

\[ V_2^{(1)} = \frac{A_2}{V_2^{(0)}} - B_{21} V_1 - B_{23} V_3^{(0)} - B_{24} V_4^{(0)} - B_{25} V_5^{(0)} \]

\[ = \frac{0.00740 + j0.00370}{1.0 - j0} - (-0.46263 + j0.00036)(1.06 + j0.00) \]

\[ - (-0.15421 + j0.00012)(1.0 + j0.0) - (-0.15421 + j0.00012)(1.0 + j0.0) \]

\[ - (-0.23131 + j0.00018)(1.0 + j0.0) \]

\[ = 1.03752 + j0.00290 \]

This value of voltage \( V_2^{(1)} \) will replace the previous value of voltage \( V_2^{(0)} \) before doing subsequent calculations of voltages.
The rate of convergence of the iterative process can be increased by applying an ACCELERATION FACTOR $\alpha$ to the approximate solution obtained. For example on hand, from the estimate $V_2^{(1)}$ we get the change in voltage

$$\Delta V_2 = V_2^{(1)} - V_2^{(0)} = (1.03752 + j0.00290) - (1.0 + j0)$$

$$= 0.03752 + j0.00290$$

The accelerated value of the bus voltage is obtained as

$$V_2^{(1)} = V_2^{(0)} + \alpha \Delta V_2$$

By assuming $\alpha = 1.4$

$$V_2^{(1)} = (1.0 + j0) + 1.4(0.03752 + j0.00290)$$

$$= 1.05253 + j0.00406$$

This new value of voltage $V_2^{(1)}$ will replace the previous value of the bus voltage $V_2^{(0)}$ and is used in the calculation of voltages for the remaining buses. In general

$$V_{k_{accl}}^{h+1} = V_k^h + \alpha (V_{k_{accl}}^{h+1} - V_k^h) \quad (2.32)$$
The process is continued for the remaining buses to complete one iteration. For the next bus 3

\[ V_{3}^{(1)} = \frac{A_3}{V_{3}^{(0)}} - B_{31} V_1 - B_{32} V_{2}^{(1)} - B_{34} V_{4}^{(0)} \]

\[ = \frac{-0.00698 - j0.00930}{1.0 - j0} - (-0.09690 + j0.00004)(1.06 + j0) \]

\[ -(-0.12920 + j0.00006)(1.05253 + j0.00406) \]

\[ -(-0.77518 + j0.00033)(1.0 + j0) \]

\[ = 1.00690 - j0.00921 \]

The accelerated value can be calculated as

\[ V_{3 \text{ accl}}^{(1)} = V_{3}^{(0)} + \infty (V_{3}^{(1)} - V_{3}^{(0)}) \]

\[ = (1.0 + j0) + 1.4 (0.00690 - j0.00921) \]

\[ = 1.00966 - j0.01289 \]
Continuing this process of calculation, at the end of first iteration, the bus voltages are obtained as

\[
\begin{align*}
V_1 &= 1.06 + j0.0 \\
V_2^{(1)} &= 1.05253 + j0.00406 \\
V_3^{(1)} &= 1.00966 - j0.01289 \\
V_4^{(1)} &= 1.01599 - j0.02635 \\
V_5^{(1)} &= 1.02727 - j0.07374
\end{align*}
\]

If \( \alpha \) and \( \beta \) are the acceleration factors for the real and imaginary components of voltages respectively, the accelerated values can be computed as

\[
\begin{align*}
e^{h+1}_{k \text{ accld}} &= e^h_k + \alpha (e^{h+1}_k - e^h_k) \\
f^{h+1}_{k \text{ accld}} &= f^h_k + \beta (f^{h+1}_k - f^h_k)
\end{align*}
\]  

(2.33)
CONVERGENCE

The iterative process must be continued until the magnitude of change of bus voltage between two consecutive iterations is less than a certain level for all bus voltages. We express this in mathematical form as

$$\Delta V_{\text{max}} = \max \text{ of } |V_k^{h+1} - V_k^h| \text{ for } k = 1, 2, \ldots, N \quad k \neq s$$

and $\Delta V_{\text{max}} < \varepsilon$

If $\varepsilon_1$ and $\varepsilon_2$ are the tolerance level for the real and imaginary parts of bus voltages respectively, then the convergence criteria will be

$$\Delta V_{\text{max}1} = \max \text{ of } |e_k^{h+1} - e_k^h|; \quad \Delta V_{\text{max}2} = \max \text{ of } |f_k^{h+1} - f_k^h|$$

for $k = 1, 2, \ldots, N \quad k \neq s$

$$\Delta V_{\text{max}1} < \varepsilon_1 \text{ and } \Delta V_{\text{max}2} < \varepsilon_2$$
For the problem under study $\varepsilon_1 = \varepsilon_2 = 0.0001$

The final bus voltages obtained after 10 iterations are given below.

\[
\begin{align*}
V_1 &= 1.06 + j0.0 \\
V_2 &= 1.04623 - j0.05126 \\
V_3 &= 1.02036 - j0.08917 \\
V_4 &= 1.01920 - j0.09504 \\
V_5 &= 1.01211 - j0.10904
\end{align*}
\]

**COMPUTATION OF LINE FLOWS AND TRANSMISSION LOSS**

Line flows can be computed from

\[
\begin{align*}
P_{km} + jQ_{km} &= V_k [(V_k^* - V_m^*) y_{km}^* + V_k^* y_{km}^*] \\
P_{12} + jQ_{12} &= V_1 [(V_1^* - V_2^*) y_{12}^* + V_1^* y_{12}^*] \\
&= (1.06 + j0)[{(1.06 - j0) - (1.04623 + j0.05126)}(5 + j15) \\
&+{(1.06 - j0)(0.0 - j0.03})] \\
&= (0.888 - j0.086)
\end{align*}
\]
Similarly
\[ P_{21} + jQ_{21} = V_2 \left[ (V_2^* - V_1^*)y_{12} + V_2^* y_{12} \right] \]
\[ = (1.04623 - j0.05126)[\{(1.04623 + j0.05126)-(1.06 - j0)\}(5 + j15) \]
\[ + \{(1.04623 + j0.05126)(0.0 - j0.03)\}] \]
\[ = (-0.874 + j0.062) \]

Power loss in line 1 – 2 is
\[ P_{L1-2} = (P_{12} + jQ_{12}) + (P_{21} + jQ_{21}) = (0.888 - j0.086) + (-0.874 + j0.062) \]
\[ = (0.014 - j0.024) \]

Power loss in other lines can be computed as
\[ P_{L1-3} = 0.012 - j0.019 \]
\[ P_{L2-3} = 0.004 - j0.033 \]
\[ P_{L2-4} = 0.004 - j0.029 \]
\[ P_{L2-5} = 0.011 + j0.002 \]
\[ P_{L3-4} = 0.0 - j0.019 \]
\[ P_{L4-5} = 0.0 - j0.051 \]

Total transmission loss = ( 0.045 - j 0.173 ) \hspace{1cm} \text{i.e.} \\
Real power transmission loss \hspace{1cm} = 4.5 \text{ MW} \\
Reactive power transmission loss \hspace{1cm} = 17.3 \text{ MVAR (Capacitive)}
COMPUTATION OF SLACK BUS POWER

Slack bus power can be determined by summing up the powers flowing out in the lines connected at the slack bus.

\[ P_s + j Q_s = (P_{13} + j Q_{13}) + (P_{12} + j Q_{12}) \]

\[ = (0.407 + j0.011) + (0.888 - j0.086) = (1.295 - j0.075) \]

Thus in this case slack bus power is

Real power = 129.5 MW

Reactive power = 7.5 MVAR (Capacitive)
**VOLTAGE CONTROLLED BUS**

In voltage controlled bus $k$ net real power injection $P_{I_k}$ and voltage magnitude $|V_k|$ are specified. Normally $Q_{I_{\text{max}}}$ and $Q_{I_{\text{min}}}$ will also be specified for voltage controlled bus. Since $Q_{I_k}$ is not known, $A_k$ given by $\frac{P_{I_k} - jQ_{I_k}}{Y_{kk}}$ cannot be calculated. An expression for $Q_{I_k}$ can be developed as shown below.

We know $I_k = \sum_{m=1}^{N} Y_{km} V_m$ and $P_{I_k} + jQ_{I_k} = V_k I_k$.

Denoting $Y_{ij} = |Y_{ij}| \angle \theta_{ij}$ and $V_i = |V_i| \angle \delta_i$, we have

$$P_{I_k} + jQ_{I_k} = |V_k| \angle \delta_k \sum_{m=1}^{N} Y_{km} |V_m| \angle -\theta_{km} - \delta_m$$

$$= |V_k| \sum_{m=1}^{N} |V_m| |Y_{km}| \angle \delta_k - \delta_m - \theta_{km}$$

Thus $Q_{I_k} = |V_k| \sum_{m=1}^{N} |V_m| Y_{km} \sin(\delta_k - \delta_m - \theta_{km})$ \hspace{1cm} (2.36)
Thus \( Q_{I_k} = |V_k| \sum_{m=1}^{N} |V_m| \ Y_{km} \ \sin(\delta_k - \delta_m - \theta_{km}) \) \hspace{1cm} \text{(2.36)}

The value of \(|V_k|\) to be used in equation (2.36) must satisfy the relation

\[ |V_k| = |V_k| \ \text{specified} \]

Because of voltage updating in the previous iteration, the voltage magnitude of the voltage controlled bus might have been deviated from the specified value. It has to be pulled back to the specified value, using the relation

Adjusted voltage \( V_k^h = |V_k| \ \text{specified} \ \angle \delta_k^h \) where \( \delta_k^h = \tan^{-1}( \frac{f_k^h}{e_k^h} ) \) taking \( V_k^h = e_k^h + j f_k^h \)

Using the adjusted voltage \( V_k^h \) as given in eqn. (2.37), net injected reactive power \( Q_{I_k}^h \) can be computed using eqn. (2.36). As long as \( Q_{I_k}^h \) falls within the range specified, \( V_k^h \) can be replaced by the Adjusted \( V_k^h \) and \( A_k \) can be computed.
In case if \( Q_{k}^{h} \) violates any one the limits specified, then \( V_{k}^{h} \) should not be replaced by Adjusted \( V_{k}^{h} \); \( Q_{k}^{h} \) is set to the limit and \( A_{k} \) can be calculated. In this case bus \( k \) is changed from P–V to P–Q type.

Once the value of \( A_{k} \) is known, further calculation to find \( V_{k}^{h+1} \) will be the same as that for P–Q bus.

Complete flow chart for power flow solution using Gauss-Seidel method is shown in Fig. 2.7. The extra calculation needed for voltage controlled bus is shown between X–X and Y–Y.
READ LINE DATA & BUS DATA
FORM Y MATRIX
ASSUME $V_k^{(0)}$, $k = 1, 2, \cdots, N$, $k \neq s$
COMPUTE $A_k$ FOR P – Q BUS
COMPUTE $B_{km}$ SET $h = 0$

SET $k = 1$ AND $\Delta V_{max} = 0.0$

$k > s$ YES

$\theta_k^h$ ADJUSTED VOLTAGE, $QI_k^h$

$QI_k^h : QI_{k,max}$ $
QI_k^h : QI_{k,min}$

REPLACE $QI_k^h$ BY $QI_{k,max}$
REPLACE $QI_k^h$ BY $QI_{k,min}$
REPLACE $V_k^h$ BY ADJ $V_k^h$

COMPUTE $V_k^{h+1}$; COMPUTE $\Delta V = [V_k^{h+1} - V_k^h]$;

$\Delta V \geq \Delta V_{max}$ YES
SET $\Delta V_{max} = \Delta V$

REPLACE $V_k^h$ BY $V_k^{h+1}$

$\Delta V < \epsilon$ YES

$V_k^h$ BY $V_k^{h+1}$

$\Delta V_{max} < \epsilon$ NO

$\Delta V \geq \Delta V_{max}$ YES

REPLACE $V_k^h$ BY $V_k^{h+1}$

$\Delta V_{max} = \Delta V$

SET $k = k + 1$

$k : N$ YES

$\Delta V_{max} < \epsilon$ YES

$\Delta V \geq \Delta V_{max}$ YES

REPLACE $V_k^h$ BY $V_k^{h+1}$

$\Delta V_{max} = \Delta V$

SET $h = h + 1$

COMPUTE LINE FLOWS, SLACK BUS POWER
PRINT THE RESULTS

STOP
Representation of off nominal tap setting transformer

A transformer with no tap setting arrangement can be represented similar to a short transmission line. However off nominal tap setting transformer which has tap setting facility at the HT side calls for different representation.

Consider a transformer 110 / 11 kV having tap setting facility. Its nominal ratio is \(10 : 1\) i.e. its nominal turns OR voltage ratio = 10. Suppose the tap is adjusted so that the turns ratio becomes \(11 : 1\). This can be thought of connecting two transformers in cascade, the first one with ratio \(11 : 10\) and the second one with the ratio \(10 : 1\) (Nominal ratio). In this case the off nominal turns ratio is \(11 / 10 = 1.1\).

Suppose the tap is adjusted so that the turns ratio becomes \(9 : 1\). This is same as two transformers in cascade, the first with a turn ratio \(9 : 10\) and the second with the ratio \(10 : 1\) (Nominal ratio). In this case off nominal turn ratio is \(9 / 10 = 0.9\).
A transformer with off nominal turn ratio can be represented by its impedance or admittance, connected in series with an ideal autotransformer as shown in Fig. 2.10.

An equivalent $\Pi$ circuit as shown in Fig. 2.11, will be useful for power flow studies. The elements of the equivalent $\Pi$ circuit can be treated in the same manner as the line elements.

Let 'a' be the turn ratio of the autotransformer. 'a' is also called as off nominal turns ratio. Usually 'a' varies from 0.85 to 1.15.

Fig. 2.10 Representation of off nominal transformer
The parameters of the equivalent $\Pi$ circuit shown in Fig. 2.11 can be derived by equating the terminal current of the transformer with the corresponding current of the equivalent $\Pi$ circuit.

It is to be noted that \( \frac{E_p}{E_t} = a \) and \( \frac{i_{tq}}{I_p} = a \) (2.86)

We are going to write \( I_p \) and \( I_q \) in terms of \( E_p \) nd \( E_q \).

\[
I_p = \frac{i_{tq}}{a} = \frac{1}{a} (E_t - E_q) y_{pq} = \frac{1}{a} \left( \frac{E_p}{a} - E_q \right) y_{pq}
\]

\[
= \frac{y_{pq}}{a^2} E_p - \frac{y_{pq}}{a} E_q
\]

Also \( I_q = (E_q - E_t) y_{pq} = (E_q - \frac{E_p}{a}) y_{pq} \) (2.88)

\[
= -\frac{y_{pq}}{a} E_p + y_{pq} E_q
\]
\[ I_p = \frac{y_{pq}}{a^2} E_p - \frac{y_{pq}}{a} E_q \]
\[ I_q = -\frac{y_{pq}}{a} E_p + y_{pq} E_q \]

Referring to the \( \Pi \) equivalent circuit

\[ I_p = (E_p - E_q)A + E_p B = (A + B)E_p - AE_q \quad \text{and} \]
\[ I_q = (E_q - E_p)A + E_q C = -AE_p + (A + C)E_q \]  
(2.89)

Comparing the coefficients of \( E_p \) and \( E_q \) in eqns. (2.87) to (2.90), we get

\[ A + B = \frac{y_{pq}}{a^2} \quad \text{Thus} \quad A = \frac{y_{pq}}{a} \]

\[ A = \frac{y_{pq}}{a} \]

\[ B = \frac{y_{pq}}{a^2} - \frac{y_{pq}}{a} = \frac{1}{a} \left( \frac{1}{a} - 1 \right) y_{pq} \]

\[ A + C = y_{pq} \]

\[ C = y_{pq} - \frac{y_{pq}}{a} = (1 - \frac{1}{a}) y_{pq} \]

Parameters \( A, B \) and \( C \) are ADMITTANCES
The equivalent $\prod$ circuit will then be as shown in Fig. 2.12

When the off nominal turns ratio is represented at bus $p$ for transformer connected between $p$ and $q$ is included, the following modifications are necessary in the bus admittance matrix.

\[
\begin{align*}
Y_{pp\_new} &= Y_{pp\_old} + \frac{y_{pq}}{a} + \frac{1}{a} \left( \frac{1}{a} - 1 \right) y_{pq} = Y_{pp\_old} + \frac{y_{pq}}{a^2} \\
Y_{qq\_new} &= Y_{qq\_old} + \frac{y_{pq}}{a} + \left( 1 - \frac{1}{a} \right) y_{pq} = Y_{qq\_old} + y_{pq} \\
Y_{pq\_new} &= Y_{pq\_old} - \frac{y_{pq}}{a} \\
Y_{qp\_new} &= Y_{qp\_old} - \frac{y_{pq}}{a}
\end{align*}
\]

(2.91)

Fig. 2.12 Equivalent circuit of off nominal transformer
Obtain the bus admittance matrix of the transmission system with the following data.

**Line data**

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Between buses</th>
<th>Line Impedance</th>
<th>HLCA</th>
<th>Off nominal turns ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 – 4</td>
<td>0.08 + j 0.37</td>
<td>j 0.007</td>
<td>---</td>
</tr>
<tr>
<td>2</td>
<td>1 – 6</td>
<td>0.123 + j 0.518</td>
<td>j 0.010</td>
<td>---</td>
</tr>
<tr>
<td>3</td>
<td>2 – 3</td>
<td>0.723 + j 1.05</td>
<td>0</td>
<td>---</td>
</tr>
<tr>
<td>4</td>
<td>2 – 5</td>
<td>0.282 + j 0.64</td>
<td>0</td>
<td>---</td>
</tr>
<tr>
<td>5</td>
<td>4 – 3</td>
<td>j 0.133</td>
<td>0</td>
<td>0.909</td>
</tr>
<tr>
<td>6</td>
<td>4 – 6</td>
<td>0.097 + j 0.407</td>
<td>j 0.0076</td>
<td>---</td>
</tr>
<tr>
<td>7</td>
<td>6 – 5</td>
<td>j 0.3</td>
<td>0</td>
<td>0.976</td>
</tr>
</tbody>
</table>

**Shunt capacitor data**

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Admittance</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>j 0.005</td>
</tr>
<tr>
<td>Line No.</td>
<td>Between buses</td>
</tr>
<tr>
<td>---------</td>
<td>--------------</td>
</tr>
<tr>
<td>1</td>
<td>1 4</td>
</tr>
<tr>
<td>2</td>
<td>1 6</td>
</tr>
<tr>
<td>3</td>
<td>2 3</td>
</tr>
<tr>
<td>4</td>
<td>2 5</td>
</tr>
<tr>
<td>5</td>
<td>4 3</td>
</tr>
<tr>
<td>6</td>
<td>4 6</td>
</tr>
<tr>
<td>7</td>
<td>6 5</td>
</tr>
</tbody>
</table>

$Y_{11} = 0.5583 - j2.582 + j 0.007 + 0.4339 - j1.8275 + j 0.01 = 0.9922 - j4.3925$

$Y_{22} = 0.4449 - j0.6461 + 0.5765 - j1.3085 = 1.0214 - j1.9546$

$Y_{33} = 0.4449 - j0.6461 - j7.5188 = 0.4449 - j8.1649$

$Y_{44} = 0.5583 - j2.582 + j 0.007 - j9.0996 + 0.5541 - j2.3249 + j 0.0076 + j0.005$

$= 1.1124 - j13.9869$

$Y_{55} = 0.5765 - j1.3085 - j3.3333 = 0.5765 - j4.6418$

$Y_{66} = 0.4339 - j1.8275 + j 0.01 + 0.5541 - j2.3249 + j0.0076 - j 3.4993 = 0.988 - j7.6341$
Answer

\[
\begin{pmatrix}
0.9922 & 0 & 0 & -0.5583 & 0 & -0.4339 \\
-\text{j}4.3925 & 0 & 0 & +\text{j}2.5820 & 0 & +\text{j}1.8275 \\
0 & 1.0214 & -0.4449 & 0 & -0.5765 & 0 \\
-\text{j}1.9546 & +\text{j}0.6461 & 0 & +\text{j}1.3085 & 0 & 0 \\
0 & -0.4449 & +0.4449 & +\text{j}8.2715 & 0 & 0 \\
+\text{j}0.6461 & -\text{j}8.1649 & +\text{j}8.2715 & 0 & 0 & 0 \\
-0.5583 & 0 & +\text{j}8.2715 & +1.1124 & 0 & -0.5541 \\
+\text{j}2.5820 & 0 & -\text{j}13.9869 & -\text{j}1.1124 & 0 & +1.1124 \\
0 & -0.5765 & 0 & 0 & +0.5765 & 0.988 \\
+\text{j}1.3085 & 0 & 0 & -\text{j}4.6418 & 0 & +0.988 \\
-0.4339 & 0 & 0 & -0.5541 & 0 & +0.988 \\
+\text{j}1.8275 & 0 & 0 & +\text{j}2.3209 & +\text{j}3.4153 & -\text{j}7.6341 \\
\end{pmatrix}
\]
% Bus admittance matrix including off nominal transformer
% Shunt parameter must be in ADMITTANCE
% Reading the line data
%
Nbus = 6; Nele = 7; Nsh = 1;
%
% Reading element data
%
Edata=[1 1 4 0.08+0.37i 0.007i 1;2 1 6 0.123+0.518i 0.01i 1;...
    3 2 3 0.723+1.05i 0 1; 4 2 5 0.282+0.64i 0 1;...
    5 4 3 0.133i 0 0.909;6 4 6 0.097+0.407i 0.0076i 1;...
    7 6 5 0.3i 0 0.976];
%
% Reading shunt data
%
Shdata = [1 4 0.005i];
%% Displaying data

```
disp(['    Sl.No            From bus           To bus              Line
Impedance          HLCA           ONTR']);
Edata
if Nsh~=0
    disp(['   Sl.No.              At bus              Shunt Admittance']);
    Shdata
end

% Formation of Ybus matrix

Ybus = zeros(Nbus,Nbus);
```
for k = 1:Nele
    p = Edata(k,2);
    q = Edata(k,3);
    yele = 1/Edata(k,4);
    Hlca = Edata(k,5);
    offa = Edata(k,6);
    offaa = offa*offa;
    Ybus(p,p) = Ybus(p,p) + yele/offaa + Hlca;
    Ybus(q,q) = Ybus(q,q) + yele + Hlca;
    Ybus(p,q) = Ybus(p,q) - yele/offa;
    Ybus(q,p) = Ybus(q,p) - yele/offa;
end
if Nsh~=0
    for i = 1:Nsh
        q = Shdata(i,2);
        yele = Shdata(i,3);
        Ybus(q,q) = Ybus(q,q) + yele;
    end
end
disp(['   *** BUS ADMITTANCE MATRIX  ***'])
Ybus
Introduction to Power Flow Analysis by Newton Raphson method

Gauss-Seidel method of solving the power flow has simple problem formulation and hence easy to explain. However, it has poor convergence characteristics. It takes large number of iterations to converge. Even for the five bus system discussed in Example 2.4, it takes 10 iterations to converge.

Newton Raphson (N.R.) method of solving power flow is based on the Newton Raphson method of solving a set of non-linear algebraic equations.

N. R. method of solving power flow problem has very good convergence characteristics. Even for large systems it takes only two to four iterations to converge.
Power flow model of Newton Raphson method

The equations describing the performance of the network in the bus admittance form is given by

\[ I = Y V \]  

(2.38)

In expanded form these equations are

\[
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{bmatrix} =
\begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1N} \\
Y_{21} & Y_{22} & \cdots & Y_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{N1} & Y_{N2} & \cdots & Y_{NN}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{bmatrix}
\]

(2.39)

Typical element of the bus admittance matrix is

\[
Y_{ij} = |Y_{ij}| \angle \theta_{ij} = |Y_{ij}| \cos \theta_{ij} + j |Y_{ij}| \sin \theta_{ij} = G_{ij} + j B_{ij}
\]

(2.40)

Voltage at a typical bus i is

\[
V_i = |V_i| \angle \delta_i = |V_i| (\cos \delta_i + j \sin \delta_i)
\]

(2.41)

The current injected into the network at bus i is given by

\[
I_i = Y_{i1} V_1 + Y_{i2} V_2 + \cdots + Y_{iN} V_N = \sum_{n=1}^{N} Y_{in} V_n
\]

(2.42)
In addition to the linear network equations given by eqn. (2.39), bus power equations should also be satisfied in the power flow problem. These bus power equations introduce non-linearity into the power flow model. The complex power entering the network at bus \( i \) is given by

\[
P_i + jQ_i = V_i \, I_i^* \tag{2.43}
\]

Bus power equations can be obtained from the above two equations (2.42) and (2.43) by eliminating the intermediate variable \( I_i \). From eqn. (2.43)

\[
P_i - jQ_i = V_i^* \, I_i = V_i^* \sum_{n=1}^{N} Y_{in} \, V_n = |V_i| \angle -\delta_i \sum_{n=1}^{N} |Y_{in}| \angle \theta_{in} |V_n| \angle \delta_n
\]

\[
= \sum_{n=1}^{N} |V_i| |V_n| |Y_{in}| / \theta_{in} + \delta_n - \delta_i
\]

Separating the real and imaginary parts, we obtain

\[
P_i = \sum_{n=1}^{N} |V_i| |V_n| |Y_{in}| \cos (\theta_{in} + \delta_n - \delta_i) \tag{2.44}
\]

\[
Q_i = -\sum_{n=1}^{N} |V_i| |V_n| |Y_{in}| \sin (\theta_{in} + \delta_n - \delta_i) \tag{2.45}
\]
\[ P_i = \sum_{n=1}^{N} |V_i||V_n||Y_{in}| \cos (\theta_{in} + \delta_n - \delta_i) \quad (2.44) \]

\[ Q_i = - \sum_{n=1}^{N} |V_i||V_n||Y_{in}| \sin (\theta_{in} + \delta_n - \delta_i) \quad (2.45) \]

The real and reactive powers obtained from the above two equations are referred as calculated powers. During the power flow calculations, their values depend on the latest bus voltages. Finally, these calculated powers should be equal to the specified powers. Thus the non-linear equations to be solved in power flow analysis are

\[ \sum_{n=1}^{N} |V_i||V_n||Y_{in}| \cos (\theta_{in} + \delta_n - \delta_i) = PI_i \quad (2.46) \]

\[ - \sum_{n=1}^{N} |V_i||V_n||Y_{in}| \sin (\theta_{in} + \delta_n - \delta_i) = QI_i \quad (2.47) \]

It is to be noted that equation (2.46) can be written for bus \( i \) only if real power injection at bus \( i \) is specified.

Similarly, equation (2.47) can be written for bus \( i \) only if reactive power injection at bus \( i \) is specified.
\[ \sum_{n=1}^{N} \left| V_i \right| \left| V_n \right| \left| Y_{in} \right| \cos(\theta_{in} + \delta_n - \delta_i) = \text{PI}_i \]  

\[ - \sum_{n=1}^{N} \left| V_i \right| \left| V_n \right| \left| Y_{in} \right| \sin(\theta_{in} + \delta_n - \delta_i) = \text{QI}_i \]  

Of the \( N \) total number of buses in the power system, let the number of P-Q buses be \( N_1 \), P-V buses be \( N_2 \). Then \( N = N_1 + N_2 + 1 \). Basic problem is to find the

i) Unknown phase angles \( \delta \) at the \( N_1 + N_2 \) number of P-Q & P-V buses and

ii) Unknown voltage magnitudes \( |V| \) at the \( N_1 \) number of P-Q buses.

Thus total number of unknown variables = \( 2N_1 + N_2 \)

We can write \( N_1 + N_2 \) real power specification equations (eqn.2.46) and \( N_1 \) reactive power specification equations (eqn.2.47).

Thus total number of equations = \( 2N_1 + N_2 \).

Therefore  \( \text{Number of equations} = \text{Number of variables} = 2N_1 + N_2 \)
Thus in power flow study, we need to solve the equations

\[ \sum_{n=1}^{N} \left| V_{i} \right| \left| V_{n} \right| \left| Y_{in} \right| \cos \left( \theta_{in} + \delta_{n} - \delta_{i} \right) = P_{I_{i}} \] (2.48)

for \( i = 1, 2, \ldots, N \)

\( i \neq s \)

and

\[ -\sum_{n=1}^{N} \left| V_{i} \right| \left| V_{n} \right| \left| Y_{in} \right| \sin \left( \theta_{in} + \delta_{n} - \delta_{i} \right) = Q_{I_{i}} \] (2.49)

for \( i = 1, 2, \ldots, N \)

\( i \neq s \)

\( i \neq P-V \) buses

for the unknown variables \( \delta_{i} \) \( i = 1, 2, \ldots, N \), \( i \neq s \) and

\[ |V_{i}| \quad i = 1, 2, \ldots, N, \quad i \neq s, \quad i \neq P-V \) buses

The unknown variables are also called as state variables.
Example 2.5

In a 9 bus system, bus 1 is the slack bus, buses 2,5 and 7 are the P-V buses. List the state variables. Also indicate the specified power injections.

Solution

Buses 3,4,6,8 and 9 are P-Q buses.

\[ \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7, \delta_8, \delta_9, |V_3|, |V_4|, |V_6|, |V_8| \text{ and } |V_9| \text{ are the state variables.} \]

\[ P_{I_2}, P_{I_3}, P_{I_4}, P_{I_5}, P_{I_6}, P_{I_7}, P_{I_8}, P_{I_9}, Q_{I_3}, Q_{I_4}, Q_{I_6}, Q_{I_8} \text{ and } Q_{I_9} \text{ are the specified power injections.} \]
Newton Raphson method of solving a set of non-linear equations

Let the non-linear equations to be solved be

\[ f_1 (x_1, x_2, \ldots, x_n) = k_1 \]

\[ f_2 (x_1, x_2, \ldots, x_n) = k_2 \]

\[ \vdots \]

\[ f_n (x_1, x_2, \ldots, x_n) = k_n \]

(2.50)

Let the initial solution be \( x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)} \)

If

\[
\begin{bmatrix}
  k_1 - f_1 (x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) \\
  k_2 - f_2 (x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) \\
  \vdots \\
  k_n - f_n (x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)})
\end{bmatrix}
= 
\begin{bmatrix}
  0 \\
  0 \\
  \vdots \\
  0
\end{bmatrix}
\]

then the solution is reached. Let us say that the solution is not reached.
Let us say that the solution is not reached. Assume $\Delta x_1, \Delta x_2, \ldots, \Delta x_n$ are the corrections required on $x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}$ respectively. Then

$$f_1[(x_1^{(0)} + \Delta x_1), (x_2^{(0)} + \Delta x_2), \ldots, (x_n^{(0)} + \Delta x_n)] = k_1$$

$$f_2[(x_1^{(0)} + \Delta x_1), (x_2^{(0)} + \Delta x_2), \ldots, (x_n^{(0)} + \Delta x_n)] = k_2$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

$$f_n[(x_1^{(0)} + \Delta x_1), (x_2^{(0)} + \Delta x_2), \ldots, (x_n^{(0)} + \Delta x_n)] = k_n$$

Each equation in the above set can be expanded by Taylor’s theorem around $x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}$. For example, the following is obtained for the first equation.

$$f_1(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) + \left. \frac{\partial f_1}{\partial x_1} \right|_0 \Delta x_1 + \left. \frac{\partial f_1}{\partial x_2} \right|_0 \Delta x_2 + \ldots + \left. \frac{\partial f_1}{\partial x_n} \right|_0 \Delta x_n + \varphi_1 = k_1$$
\[ f_1(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) + \left. \frac{\partial f_1}{\partial x_1} \right|_0 \Delta x_1 + \left. \frac{\partial f_1}{\partial x_2} \right|_0 \Delta x_2 + \ldots + \left. \frac{\partial f_1}{\partial x_n} \right|_0 \Delta x_n + \phi_1 = k_1 \]

where \( \phi_1 \) is a function of higher derivatives of \( f_1 \) and higher powers of \( \Delta x_1, \Delta x_2, \ldots, \Delta x_n \). Neglecting \( \phi_1 \) and also following the same for other equations, we get

\[ f_1(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) + \left. \frac{\partial f_1}{\partial x_1} \right|_0 \Delta x_1 + \left. \frac{\partial f_1}{\partial x_2} \right|_0 \Delta x_2 + \ldots + \left. \frac{\partial f_1}{\partial x_n} \right|_0 \Delta x_n = k_1 \]

\[ f_2(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) + \left. \frac{\partial f_2}{\partial x_1} \right|_0 \Delta x_1 + \left. \frac{\partial f_2}{\partial x_2} \right|_0 \Delta x_2 + \ldots + \left. \frac{\partial f_2}{\partial x_n} \right|_0 \Delta x_n = k_2 \]

\[ \vdots \]

\[ f_n(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) + \left. \frac{\partial f_n}{\partial x_1} \right|_0 \Delta x_1 + \left. \frac{\partial f_n}{\partial x_2} \right|_0 \Delta x_2 + \ldots + \left. \frac{\partial f_n}{\partial x_n} \right|_0 \Delta x_n = k_n \]
The matrix form of equations (2.52) is

\[
\begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & 0 & \cdots & 0 \\
0 & \frac{\partial f_1}{\partial x_2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \frac{\partial f_1}{\partial x_n}
\end{bmatrix}
\begin{bmatrix}
\Delta x_1 \\
\Delta x_2 \\
\vdots \\
\Delta x_n
\end{bmatrix} = 
\begin{bmatrix}
k_1 - f_1(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) \\
k_2 - f_2(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) \\
\vdots \\
k_n - f_n(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)})
\end{bmatrix}
\] (2.53)

The above equation can be written in a compact form as

\[
F'(X^{(0)}) \Delta X = K - F(X^{(0)})
\] (2.54)

This set of linear equations need to be solved for the correction vector

\[
\Delta X = 
\begin{bmatrix}
\Delta x_1 \\
\Delta x_2 \\
\vdots \\
\Delta x_n
\end{bmatrix}
\]
\[ F'(X^{(0)}) \Delta X = K - F(X^{(0)}) \]  \hspace{1cm} (2.54) 

In eqn. (2.54) \( F'(X^{(0)}) \) is called the **JACOBIAN MATRIX** and the vector \( K - F(X^{(0)}) \) is called the **ERROR VECTOR**. The Jacobian matrix is also denoted as \( J \).

Solving eqn. (2.54) for \( \Delta X \)

\[ \Delta X = \left[ F'(X^{(0)}) \right]^{-1} \left[ K - F(X^{(0)}) \right] \]  \hspace{1cm} (2.55) 

Then the improved estimate is

\[ X^{(1)} = X^{(0)} + \Delta X \]

Generalizing this, for \((h+1)^{th}\) iteration

\[ X^{(h+1)} = X^{(h)} + \Delta X \quad \text{where} \]

\[ \Delta X = \left[ F'(X^{(h)}) \right]^{-1} \left[ K - F(X^{(h)}) \right] \]  \hspace{1cm} (2.56) 

\[ \Delta X \] is the solution of \[ F'(X^{(h)}) \Delta X = K - F(X^{(h)}) \]  \hspace{1cm} (2.57) 

\[ \text{i.e. } \Delta X \text{ is the solution of } F'(X^{(h)}) \Delta X = K - F(X^{(h)}) \]  \hspace{1cm} (2.58)
Thus the solution procedure to solve $F(X) = K$ is as follows:

(i) Calculate the error vector $K - F(X^{(h)})$

If the error vector $\approx$ zero, convergence is reached; otherwise formulate

$$F'(X^{(h)}) \Delta X = K - F(X^{(h)})$$

(ii) Solve for the correction vector $\Delta X$

(iii) Update the solution as

$$X^{(h+1)} = X^{(h)} + \Delta X$$

Values of the correction vector can also be used to test for convergence.

It is to be noted that Error vector = specified vector – vector of calculated values
Example 2.6

Using Newton-Raphson method, solve for $x_1$ and $x_2$ of the non-linear equations

\[ 4 \ x_2 \ \sin \ x_1 = - 0.6 \]

\[ 4 \ x_2^2 - 4 \ x_2 \ \cos \ x_1 = - 0.3 \]

Choose the initial solution as $x_1^{(0)} = 0$ rad. and $x_2^{(0)} = 1$. Take the precision index on error vector as $10^{-3}$.

Solution

Errors are calculated as

\[
- 0.6 - (4 \ x_2 \ \sin \ x_1) \bigg|_{x_1 = 0, \ x_2 = 1} = - 0.6
\]

\[
- 0.3 - (4 \ x_2^2 - 4 \ x_2 \ \cos \ x_1) \bigg|_{x_1 = 0, \ x_2 = 1} = - 0.3
\]

The error vector $\begin{bmatrix} -0.6 \\ -0.3 \end{bmatrix}$ is not small.
It is noted that \[ f_1 = 4 \, x_2 \, \sin \, x_1 \]

\[ f_2 = 4 \, x_2^2 - 4 \, x_2 \, \cos \, x_1 \]

Jacobian matrix is: \[
J = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2}
\end{bmatrix} = \begin{bmatrix} 4x_2 \, \cos x_1 & 4 \, \sin x_1 \\ 4x_2 \, \sin x_1 & 8x_2 - 4 \, \cos x_1 \end{bmatrix}
\]

Substituting the latest values of state variables \[ x_1 = 0 \] and \[ x_2 = 1 \]

\[ J = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \]; Its inverse is \[ J^{-1} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix} \]

Correction vector is calculated as

\[
\begin{bmatrix}
\Delta x_1 \\
\Delta x_2
\end{bmatrix} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix} \begin{bmatrix} -0.6 \\ -0.3 \end{bmatrix} = \begin{bmatrix} -0.150 \\ -0.075 \end{bmatrix}
\]

The state vector is updated as

\[
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.150 \\ -0.075 \end{bmatrix} = \begin{bmatrix} -0.150 \\ 0.925 \end{bmatrix}
\]

This completes the first iteration.
Errors are calculated as

\[-0.6 - (4 \ x_2 \ \sin \ x_1) \quad x_1 = -0.15 \ \text{rad.} \quad x_2 = 0.925 \quad = -0.047079\]

\[-0.3 - (4 \ x_2^2 - 4 \ x_2 \cos \ x_1) \quad x_1 = -0.15 \ \text{rad.} \quad x_2 = 0.925 \quad = -0.064047\]

The error vector \[
\begin{bmatrix}
-0.04709 \\
-0.064047
\end{bmatrix}
\]
is not small.

Jacobian matrix is: \[
J = \begin{bmatrix}
4 x_2 \ \cos x_1 & 4 \ \sin x_1 \\
4 x_2 \ \sin x_1 & 8 x_2 - 4 \ \cos x_1
\end{bmatrix}
\quad x_1 = -0.15 \ \text{rad.} \quad x_2 = 0.925
\]

\[
= \begin{bmatrix}
3.658453 & -0.597753 \\
-0.552921 & 3.444916
\end{bmatrix}
\]
Correction vector is calculated as

$$
\Delta x = \begin{bmatrix}
\Delta x_1 \\
\Delta x_2
\end{bmatrix} = \begin{bmatrix}
3.658453 & -0.597753 \\
-0.552921 & 3.444916
\end{bmatrix} \begin{bmatrix}
-0.047079 \\
-0.064047
\end{bmatrix} = \begin{bmatrix}
-0.016335 \\
-0.021214
\end{bmatrix}
$$

The state vector is updated as

$$
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = \begin{bmatrix}
-0.150 \\
0.925
\end{bmatrix} + \begin{bmatrix}
-0.016335 \\
-0.021214
\end{bmatrix} = \begin{bmatrix}
-0.166335 \\
0.903786
\end{bmatrix}
$$

This completes the second iteration.

Errors are calculated as

$$
- 0.6 - (4 \ x_2 \ \sin \ x_1) \bigg|_{x_1 = -0.166335 \ \text{rad.}}^{x_2 = 0.903786} = - 0.001444
$$

$$
- 0.3 - (4 \ x_2^2 - 4 \ x_2 \ \cos \ x_1) \bigg|_{x_1 = -0.166335 \ \text{rad.}}^{x_2 = 0.903786} = - 0.002068
$$

Still error vector exceeds the precision index.
Jacobian matrix is:

\[
J = \begin{bmatrix}
4x_2 \cos x_1 & 4 \sin x_1 \\
4x_2 \sin x_1 & 8x_2 - 4 \cos x_1
\end{bmatrix}
\]

\[x_1 = -0.166335 \text{ rad.}\]
\[x_2 = 0.903786\]

\[
= \begin{bmatrix}
3.565249 & -0.662276 \\
-0.598556 & 3.285495
\end{bmatrix}
\]

Correction vector is calculated as

\[
\begin{bmatrix}
\Delta x_1 \\
\Delta x_2
\end{bmatrix}
= \begin{bmatrix}
3.565249 & -0.662276 \\
-0.598556 & 3.285495
\end{bmatrix}^{-1}
\begin{bmatrix}
-0.001444 \\
-0.002068
\end{bmatrix}
= \begin{bmatrix}
-0.000540 \\
-0.000728
\end{bmatrix}
\]

The state vector is updated as

\[
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= \begin{bmatrix}
-0.166335 \\
0.903786
\end{bmatrix}
+ \begin{bmatrix}
-0.000540 \\
-0.000728
\end{bmatrix}
= \begin{bmatrix}
-0.166875 \\
0.903058
\end{bmatrix}
\]

\[0.903786 \text{ rad.} 0.166335 \approx 0.903786 \text{ rad.} 0.166335 \]

\[\approx - \]

\[-1\]
Errors are calculated as

\[- 0.6 - (4 \ x_2 \ \sin \ x_1) \bigg| x_1 = -0.166875 \text{ rad.} \]

\[- 0.3 - (4 \ x_2^2 - 4 \ x_2 \ \cos \ x_1) \bigg| x_2 = 0.903058 \text{ } = \text{ } -0.000002 \]

Errors are less than $10^{-3}$.

The final values of $x_1$ and $x_2$ are $-0.166875$ rad. and $0.903058$ respectively.

The results can be checked by substituting the solution in the original equations:

$4 \ x_2 \ \sin \ x_1 = -0.6$

$4 \ x_2^2 - 4 \ x_2 \ \cos \ x_1 = -0.3$
In this example, we have actually solved our first power flow problem by N.R. method. This is because the two non-linear equations of this example are the power flow model of the simple system shown in Fig. 2.8.

Fig. 2.8 Two bus power flow model

Here bus 1 is the slack bus with its voltage $|V_1| \angle \delta_1 = 1.0 \angle 0^0$ p.u. Further $x_1$ represents the angle $\delta_2$ and $x_2$ represents the voltage magnitude $|V_2|$ at bus 2.

We now concentrate on the application of Newton-Raphson procedure in the power flow studies.
Power flow solution by Newton Raphson method

As discussed earlier, taking the bus voltages and line admittances in polar form, in power flow study we need to solve the non-linear equations

\[ \sum_{n=1}^{N} |V_i||V_n||Y_{in}| \cos (\theta_{in} + \delta_n - \delta_i) = PI_i \quad (2.59) \]

\[ - \sum_{n=1}^{N} |V_i||V_n||Y_{in}| \sin (\theta_{in} + \delta_n - \delta_i) = QI_i \quad (2.60) \]

Separating the term with \( n = i \) we get

\[ |V_i|^2 G_{ii} + \sum_{n=1}^{N} |V_i||V_n||Y_{in}| \cos (\theta_{in} + \delta_n - \delta_i) = PI_i \quad (2.61) \]

\[ - |V_i|^2 B_{ii} - \sum_{n=1}^{N} |V_i||V_n||Y_{in}| \sin (\theta_{in} + \delta_n - \delta_i) = QI_i \quad (2.62) \]
In a compact form, the above non-linear equations can be written as

\[ P(\delta, V) = P_I \]  

\[ Q(\delta, V) = Q_I \]  

On linearization, we get

\[
\begin{bmatrix}
\frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} \\
\frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta |V|
\end{bmatrix}
= 
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]  

(2.65)

where

\[ \Delta P = P_I - \text{calculated value of } P(\delta, |V|) \text{ corresponding to the present solution.} \]

\[ \Delta Q = Q_I - \text{calculated value of } Q(\delta, |V|) \text{ corresponding to the present solution.} \]
To bring symmetry in the elements of the coefficient matrix, \( \frac{\Delta V}{V} \) is taken as problem variable in place of \( \Delta |V| \). Then eqn. (2.65) changes to

\[
\begin{bmatrix}
\frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} & \frac{\partial P}{\partial V} \\
\frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|} & \frac{\partial Q}{\partial V}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta |V| \\
\Delta V
\end{bmatrix}
=
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]

(2.66)

In symbolic form, the above equation can be written as

\[
\begin{bmatrix}
H & N \\
M & L
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta |V| \\
\Delta V
\end{bmatrix}
=
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]

(2.67)
The matrix $\begin{bmatrix} H & N \\ M & L \end{bmatrix}$ is known as JACOBIAN matrix.

The dimensions of the sub-matrices will be as follows:

- $H = (N_1 + N_2) \times (N_1 + N_2)$
- $N = (N_1 + N_2) \times N_1$
- $M = N_1 \times (N_1 + N_2)$ and
- $L = N_1 \times N_1$

where $N_1$ is the number of P-Q buses and $N_2$ is the number of P-V buses.

Consider a 4 bus system having bus 1 as slack bus, buses 2 and 3 as P-Q buses and bus 4 as P-V bus for which real power injections $P_{I_2}, P_{I_3}, P_{I_4}$ and reactive power injections $Q_{I_2}, Q_{I_3}$ are specified. Noting that $\delta_2, \delta_3, \delta_4, |V_2|$ and $|V_3|$ are the problem variables, linear equations that are to be solved in each iteration will be
\[
\begin{bmatrix}
\delta_2 & \delta_3 & \delta_4 & | V_2 | & | V_3 | \\
\hline
P_2 & \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial \delta_4} & \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_3} | V_2 | & \frac{\partial P_2}{\partial V_3} | V_3 | & \Delta \delta_2 & \Delta P_2 \\
\hline
P_3 & \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial \delta_4} & \frac{\partial P_3}{\partial V_2} & \frac{\partial P_3}{\partial V_3} | V_2 | & \frac{\partial P_3}{\partial V_3} | V_3 | & \Delta \delta_3 & \Delta P_3 \\
\hline
P_4 & \frac{\partial P_4}{\partial \delta_2} & \frac{\partial P_4}{\partial \delta_3} & \frac{\partial P_4}{\partial \delta_4} & \frac{\partial P_4}{\partial V_2} & \frac{\partial P_4}{\partial V_3} | V_2 | & \frac{\partial P_4}{\partial V_3} | V_3 | & \Delta \delta_4 & \Delta P_4 \\
\hline
Q_2 & \frac{\delta Q_2}{\partial \delta_2} & \frac{\delta Q_2}{\partial \delta_3} & \frac{\delta Q_2}{\partial \delta_4} & \frac{\delta Q_2}{\partial V_2} & \frac{\delta Q_2}{\partial V_3} | V_2 | & \frac{\delta Q_2}{\partial V_3} | V_3 | & \Delta | V_2 | | V_2 | & \Delta Q_2 \\
\hline
Q_3 & \frac{\delta Q_3}{\partial \delta_2} & \frac{\delta Q_3}{\partial \delta_3} & \frac{\delta Q_3}{\partial \delta_4} & \frac{\delta Q_3}{\partial V_2} & \frac{\delta Q_3}{\partial V_3} | V_2 | & \frac{\delta Q_3}{\partial V_3} | V_3 | & \Delta | V_3 | | V_3 | & \Delta Q_3
\end{bmatrix}
\]

\[2.68\]
The following is the solution procedure for N.R. method of power flow analysis.

1. Read the line data and bus data; construct the bus admittance matrix.

2. Set $k = 0$. Assume a starting solution. Usually a FLAT START is assumed in which all the unknown phase angles are taken as zero and unknown voltage magnitudes are taken as 1.0 p.u.
3 Compute the mismatch powers i.e. the error vector. If the elements of error vector are less than the specified tolerance, the problem is solved and hence go to Step 7; otherwise proceed to Step 4.

4 Compute the elements of sub-matrices $H, N, M$ and $L$. Solve

$$
\begin{bmatrix}
H & N \\
M & L
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta V
\end{bmatrix}_k
= 
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}_k
$$

for

$$
\begin{bmatrix}
\Delta \delta \\
\Delta V
\end{bmatrix}
$$

5 Update the solution as

$$
\begin{bmatrix}
\delta \\
|V|
\end{bmatrix}_{k+1}
= 
\begin{bmatrix}
\delta \\
|V|
\end{bmatrix}_k 
+ 
\begin{bmatrix}
\Delta \delta \\
\Delta |V|
\end{bmatrix}
$$

6 Set $k = k + 1$ and go to Step 3.

7 Calculate line flows, transmission loss and slack bus power. Print the results and STOP.
Calculation of elements of Jacobian matrix

We know that the equations that are to be solved are

\[ |V_i|^2 \ G_{ii} + \sum_{n=1\atop n \neq i}^{N} |V_i| \ |V_n| \ |Y_{in}| \ \cos (\theta_{in} + \delta_n - \delta_i) = P_{I_i} \]  \hspace{1cm} (2.69)

\[ -|V_i|^2 \ B_{ii} - \sum_{n=1\atop n \neq i}^{N} |V_i| \ |V_n| \ |Y_{in}| \ \sin (\theta_{in} + \delta_n - \delta_i) = Q_{I_i} \]  \hspace{1cm} (2.70)

i.e.

\[ P_i (\delta, |V|) = P_{I_i} \]  \hspace{1cm} (2.71)

\[ Q_i (\delta, |V|) = Q_{I_i} \]  \hspace{1cm} (2.72)

The suffix i should take necessary values.

Jacobian matrix is

\[
\begin{bmatrix}
H & N \\
M & L
\end{bmatrix}
\]

where

\[ H = \frac{\partial P}{\partial \delta}; \quad N = \frac{\partial P}{\partial |V|} |V|; \quad M = \frac{\partial Q}{\partial \delta} \quad \text{and} \quad L = \frac{\partial Q}{\partial |V|} |V| \]
Here

\[ P_i = |V_i|^2 G_{ii} + \sum_{\substack{n=1 \atop n \neq i}}^N |V_i||V_n||Y_{in}| \cos (\theta_{in} + \delta_n - \delta_i) \quad (2.73) \]

\[ Q_i = -|V_i|^2 B_{ii} - \sum_{\substack{n=1 \atop n \neq i}}^N |V_i||V_n||Y_{in}| \sin (\theta_{in} + \delta_n - \delta_i) \quad (2.74) \]

**Diagonal elements:**

\[ H_{ii} = \frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{n=1 \atop n \neq i}}^N |V_i||V_n||Y_{in}| \sin (\theta_{in} + \delta_n - \delta_i) = -Q_i - |V_i|^2 B_{ii} \quad (2.75) \]

\[ N_{ii} = \frac{\partial P_i}{\partial V_i} |V_i| = 2|V_i|^2 G_{ii} + \sum_{\substack{n=1 \atop n \neq i}}^N |V_i||V_n||Y_{in}| \cos (\theta_{in} + \delta_n - \delta_i) \]

\[ = P_i + |V_i|^2 G_{ii} \quad (2.76) \]

\[ M_{ii} = \frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{n=1 \atop n \neq i}}^N |V_i||V_n||Y_{in}| \cos (\theta_{in} + \delta_n - \delta_i) = P_i - |V_i|^2 G_{ii} \quad (2.77) \]

\[ L_{ii} = \frac{\partial Q_i}{\partial V_i} |V_i| = -2|V_i|^2 B_{ii} - \sum_{\substack{n=1 \atop n \neq i}}^N |V_i||V_n||Y_{in}| \sin (\theta_{in} + \delta_n - \delta_i) \]

\[ = Q_i - |V_i|^2 B_{ii} \quad (2.78) \]
Off-diagonal elements:

We know that

\[ P_i = |V_i|^2 G_{ii} + \sum_{n=1}^{N} \left| V_i \right| \left| V_n \right| Y_{in} \cos (\theta_{in} + \delta_n - \delta_i) \]  

(2.79)

\[ Q_i = -|V_i|^2 B_{ii} - \sum_{n=1}^{N} \left| V_i \right| \left| V_n \right| Y_{in} \sin (\theta_{in} + \delta_n - \delta_i) \]  

(2.80)

\[ H_{ij} = \frac{\partial P_i}{\partial \delta_j} = - \left| V_i \right| \left| V_j \right| Y_{ij} \sin (\theta_{ij} + \delta_j - \delta_i) \]  

(2.81)

\[ N_{ij} = \frac{\partial P_i}{\partial |V_j|} = \left| V_i \right| \left| V_j \right| Y_{ij} \cos (\theta_{ij} + \delta_j - \delta_i) \]  

(2.82)

\[ M_{ij} = \frac{\partial Q_i}{\partial \delta_j} = - \left| V_i \right| \left| V_j \right| Y_{ij} \cos (\theta_{ij} + \delta_j - \delta_i) \]  

(2.83)

\[ L_{ij} = \frac{\partial Q_i}{\partial |V_j|} = - \left| V_i \right| \left| V_j \right| Y_{ij} \sin (\theta_{ij} + \delta_j - \delta_i) \]  

(2.84)
Summary of formulae

\[ H_{ii} = -Q_i - |V_i|^2 B_{ii} \]

\[ N_{ii} = P_i + |V_i|^2 G_{ii} \]

\[ M_{ii} = P_i - |V_i|^2 G_{ii} \]

\[ L_{ii} = Q_i - |V_i|^2 B_{ii} \]

\[ H_{ij} = -|V_i||V_j||Y_{ij}| \sin (\theta_{ij} + \delta_j - \delta_i) \]

\[ N_{ij} = |V_i||V_j||Y_{ij}| \cos (\theta_{ij} + \delta_j - \delta_i) \]

\[ M_{ij} = -|V_i||V_j||Y_{ij}| \cos (\theta_{ij} + \delta_j - \delta_i) \]

\[ L_{ij} = -|V_i||V_j||Y_{ij}| \sin (\theta_{ij} + \delta_j - \delta_i) \]

Flow chart for N.R. method of power flow solution is shown below.
START

READ LINE and BUS DATA
COMPUTE Y MATRIX
ASSUME FLAT START

FOR ALL P-V BUSES COMPUTE $Q_i$
IF $Q_i$ VIOLATES THE LIMITS SET
$Q_i = Q_i \text{ Limit}$ AND TREAT BUS $i$ AS A P-Q BUS

COMPUTE MISMATCH POWERS $\Delta P$ & $\Delta Q$

COMPUTE MATRICES $H, N, M & L$
FORM

\[
\begin{bmatrix}
H & N \\
M & L
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta \|V\|
\end{bmatrix}
= \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix};
\text{SOLVE FOR}
\begin{bmatrix}
\Delta \delta \\
\Delta \|V\|
\end{bmatrix}
\]

AND UPDATE

\[
\begin{bmatrix}
\delta \\
\|V\|
\end{bmatrix}_{k+1}
= \begin{bmatrix}
\delta \\
\|V\|
\end{bmatrix}_k + \begin{bmatrix}
\Delta \delta \\
\Delta \|V\|
\end{bmatrix}
\]

SET $k = k + 1$

COMPUTE LINE FLOWS, TRANSMISSION LOSS & SLACK BUS POWER
PRINT THE RESULTS

STOP
Example 2.7

Perform power flow analysis for the power system with the data given below, using Newton Raphson method, and obtain the bus voltages.

Line data (p.u. quantities)

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Between buses</th>
<th>Line impedances</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1–2</td>
<td>0 + j0.1</td>
</tr>
<tr>
<td>2</td>
<td>2–3</td>
<td>0 + j0.2</td>
</tr>
<tr>
<td>3</td>
<td>1–3</td>
<td>0 + j0.2</td>
</tr>
</tbody>
</table>

Bus data (p.u. quantities)

| Bus No | Type  | Generator | Load | | V | δ | Q_{min} | Q_{max} |
|--------|-------|-----------|------|---|---|---------|---------|
| 1      | Slack | ---       | 0    | 0 | 1.0 | 0       | ---     | ---     |
| 2      | P - V | 1.8184    | 0    | --- | 1.1 | ---     | 0       | 3.5     |
| 3      | P - Q | 0         | 1.2517 | 1.2574 | --- | ---     | ---     | ---     |
Solution

The bus admittance matrix can be obtained as

\[
Y = \begin{bmatrix}
1 & 2 & 3 \\
1 & -j15 & j10 & j5 \\
2 & j10 & -j15 & j5 \\
3 & j5 & j5 & -j10 \\
\end{bmatrix}
\]

\[
G = [0]; \quad B = \begin{bmatrix}
-15 & 10 & 5 \\
10 & -15 & 5 \\
5 & 5 & -10 \\
\end{bmatrix}
\]

\[
|Y| = \begin{bmatrix}
1 & 2 & 3 \\
1 & 15 & 10 & 5 \\
2 & 10 & 15 & 5 \\
3 & 5 & 5 & 10 \\
\end{bmatrix}
\]

\[
\theta = \begin{bmatrix}
1 & 2 & 3 \\
1 & -90^\circ & 90^\circ & 90^\circ \\
2 & 90^\circ & -90^\circ & 90^\circ \\
3 & 90^\circ & 90^\circ & -90^\circ \\
\end{bmatrix}
\]

\[
\Pi_2 = 1.8184 \\
\Pi_3 = -1.2517 \\
QI_3 = -1.2574
\]

In this problem and unknown quantities = \[
\begin{bmatrix}
\delta_2 \\
\delta_3 \\
V_3 \\
\end{bmatrix}
\]
With flat start \( V_1 = 1.0 \angle 0^0 \)

\[ V_2 = 1.1 \angle 0^0 \]

\[ V_3 = 1.0 \angle 0^0 \]

We know that

\[
P_i = |V_i|^2 G_{ii} + \sum_{n=1, n \neq i}^{N} |V_i| |V_n| |Y_{in}| \cos (\theta_{in} + \delta_n - \delta_i)
\]

\[
Q_i = -|V_i|^2 B_{ii} - \sum_{n=1, n \neq i}^{N} |V_i| |V_n| |Y_{in}| \sin (\theta_{in} + \delta_n - \delta_i)
\]

Substituting the values of bus admittance parameters, expressions for \( P_2, P_3 \) and \( Q_3 \) are obtained as follows

\[
P_2 = |V_2|^2 G_{22} + |V_2||V_1| Y_{21} \cos (\theta_{21} + \delta_1 - \delta_2) + |V_2||V_3| Y_{23} \cos (\theta_{23} + \delta_3 - \delta_2)
\]

\[
= 0 + 10 |V_2||V_1| \cos (90 + \delta_1 - \delta_2) + 5 |V_2||V_3| \cos (90 + \delta_3 - \delta_2)
\]

\[
= -10 |V_2||V_1| \sin (\delta_1 - \delta_2) - 5 |V_2||V_3| \sin (\delta_3 - \delta_2)
\]
Similarly

\[ P_3 = -5 \left| V_3 \right| \left| V_1 \right| \sin (\delta_1 - \delta_3) - 5 \left| V_3 \right| \left| V_2 \right| \sin (\delta_2 - \delta_3) \]

Likewise

\[ Q_3 = - \left| V_3 \right|^2 B_{33} - \left| V_3 \right| \left| V_1 \right| \left| Y_{31} \right| \sin (90 + \delta_1 - \delta_3) - \left| V_3 \right| \left| V_2 \right| \left| Y_{32} \right| \sin (90 + \delta_2 - \delta_3) \]

\[ = 10 \left| V_3 \right|^2 - 5 \left| V_3 \right| \left| V_1 \right| \cos (\delta_1 - \delta_3) - 5 \left| V_3 \right| \left| V_2 \right| \cos (\delta_2 - \delta_3) \]

Iteration starts:

To check whether bus 2 will remain as P-V bus, \( Q_2 \) need to be calculated.

\[ Q_2 = 15 \left| V_2 \right|^2 - 10 \left| V_2 \right| \left| V_1 \right| \cos (\delta_1 - \delta_2) - 5 \left| V_2 \right| \left| V_3 \right| \cos (\delta_3 - \delta_2) \]

\[ = (15 \times 1.1 \times 1.1) - (10 \times 1.1 \times 1 \times 1) - (5 \times 1.1 \times 1 \times 1) = 1.65 \]

This lies within the Q limits. Thus bus 2 remains as P-V bus.
Since $\delta_1 = \delta_2 = \delta_3 = 0$,

we get $P_2 = P_3 = 0$

$Q_3 = (10 \times 1 \times 1) - (5 \times 1 \times 1) - (5 \times 1 \times 1.1) = -0.5$

Mismatch powers are:

$\Delta P_2 = P_{I_2} - P_2 = 1.8184 - 0 = 1.8184$

$\Delta P_3 = P_{I_3} - P_3 = -1.2517 - 0 = -1.2517$

$\Delta Q_3 = Q_{I_3} - Q_3 = -1.2574 + 0.5 = -0.7574$

Linear equations to be solved are:

$$\begin{bmatrix}
\delta_2 & \delta_3 \\
H_{22} & H_{23} \\
H_{32} & H_{33}
\end{bmatrix}
\begin{bmatrix}
V_3 \\
N_{23} \\
N_{33}
\end{bmatrix}
= 
\begin{bmatrix}
\Delta \delta_2 \\
\Delta \delta_3 \\
\Delta |V_3|
\end{bmatrix}
= 
\begin{bmatrix}
\Delta P_2 \\
\Delta P_3 \\
\Delta Q_3
\end{bmatrix}$$
Summary of formulae

\[
H_{ii} = -Q_i - |V_i|^2 B_{ii}
\]

\[
N_{ii} = P_i + |V_i|^2 G_{ii}
\]

\[
M_{ii} = P_i - |V_i|^2 G_{ii}
\]

\[
L_{ii} = Q_i - |V_i|^2 B_{ii}
\]

\[
H_{ij} = -|V_i||V_j||Y_{ij}| \sin (\theta_{ij} + \delta_j - \delta_i)
\]

\[
N_{ij} = |V_i||V_j||Y_{ij}| \cos (\theta_{ij} + \delta_j - \delta_i)
\]

\[
M_{ij} = -|V_i||V_j||Y_{ij}| \cos (\theta_{ij} + \delta_j - \delta_i)
\]

\[
L_{ij} = -|V_i||V_j||Y_{ij}| \sin (\theta_{ij} + \delta_j - \delta_i)
\]

(2.85)
For this problem, since \( G_{ii} \) are zero and \( \theta_{ij} \) are 90°

\[
H_{ii} = -Q_i - |V_i|^2 B_{ii} \\
N_{ii} = P_i; \quad M_{ii} = P_i \\
L_{ii} = Q_i - |V_i|^2 B_{ii}
\]

\[
H_{ij} = - |V_i| |V_j| \gamma_{ij} \cos(\delta_j - \delta_i) \\
N_{ij} = - |V_i| |V_j| \gamma_{ij} \sin(\delta_j - \delta_i) \\
M_{ij} = |V_i| |V_j| \gamma_{ij} \sin(\delta_j - \delta_i)
\]

\[
H_{22} = -Q_2 - |V_2|^2 B_{22} = -1.65 + (1.1 \times 1.1 \times 15) = 16.5
\]

\[
H_{33} = -Q_3 - |V_3|^2 B_{33} = 0.5 + 10 = 10.5
\]

\[
N_{33} = P_3 = 0; \quad M_{33} = P_3 = 0
\]

\[
L_{33} = Q_3 - |V_3|^2 B_{33} = -0.5 + 10 = 9.5
\]

\[
H_{23} = - |V_2| |V_3| |Y_{23}| \cos(\delta_3 - \delta_2) = -1.1 \times 1 \times 5 \times 1 = -5.5 \quad \text{and} \quad H_{32} = -5.5
\]

\[
N_{23} = - |V_2| |V_3| |Y_{23}| \sin(\delta_3 - \delta_2) = 0
\]

\[
M_{23} = |V_3| |V_2| |Y_{32}| \sin(\delta_2 - \delta_3) = 0
\]
Thus
\[
\begin{bmatrix}
16.5 & -5.5 & 0 \\
-5.5 & 10.5 & 0 \\
0 & 0 & 9.5
\end{bmatrix}
\begin{bmatrix}
\Delta\delta_2 \\
\Delta\delta_3 \\
\Delta|V_3|
\end{bmatrix}
= 
\begin{bmatrix}
1.8184 \\
-1.2517 \\
-0.7574
\end{bmatrix}
\]

Solving the above
\[
\begin{bmatrix}
\Delta\delta_2 \\
\Delta\delta_3 \\
\Delta|V_3|
\end{bmatrix}
= 
\begin{bmatrix}
0.08538 \\
-0.07449 \\
-0.0797
\end{bmatrix}
\]

Therefore
\[
\delta_2^{(1)} = 0 + 0.08538 = 0.08538 \text{ rad.} = 4.89^0
\]
\[
\delta_3^{(1)} = 0 - 0.07449 = -0.07449 \text{ rad.} = -4.27^0
\]
\[
|V_3|^{(1)} = 1.0 - 0.0797 = 0.9203
\]

Thus
\[
V_1 = 1.0 \angle 0^0
\]
\[
V_2 = 1.1 \angle 4.89^0
\]
\[
V_3 = 0.9203 \angle -4.27^0
\]

This completes the first iteration.
Second iteration:

\[ Q_2 = (15 \times 1.1 \times 1.1) - (10 \times 1.1 \times 1.0 \cos 4.89^0) - (5 \times 1.1 \times 0.9203 \cos 9.16^0) \]

\[ = 2.1929 \]

This is within the limits. Bus 2 remains as P-V bus.

\[ P_2 = (10 \times 1.1 \times 1.0 \sin 4.89^0) + (5 \times 1.1 \times 0.9203 \sin 9.16^0) = 1.7435 \]

\[ P_3 = -(5 \times 0.9203 \times 1.0 \sin 4.27^0) - (5 \times 0.9203 \times 1.1 \sin 9.16^0) = -1.1484 \]

\[ Q_3 = 10 \times 0.9203 \times 0.9203 - (5 \times 0.9203 \times 1.0 \cos 4.27^0) - (5 \times 0.9203 \times 1.1 \cos 9.16^0) \]

\[ = -1.1163 \]

\[ \Delta P_2 = 1.8184 - 1.7435 = 0.0749 \]

\[ \Delta P_3 = -1.2517 + 1.1484 = -0.1033 \]

\[ \Delta Q_3 = -1.2574 + 1.1163 = -0.1444 \]
The linear equations are

\[
\begin{bmatrix}
15.9571 & -4.9971 & 0.8058 \\
-4.9971 & 9.5858 & -1.1484 \\
0.8058 & -1.1484 & 7.3532
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_2 \\
\Delta \delta_3 \\
\Delta |V_3|
\end{bmatrix}
= 
\begin{bmatrix}
0.0749 \\
-0.1033 \\
-0.1444
\end{bmatrix}
\]
Its solution is

\[
\begin{bmatrix}
\Delta \delta_2 \\
\Delta \delta_3 \\
\Delta |V_3| \\
|V_3|
\end{bmatrix} =
\begin{bmatrix}
0.001914 \\
-0.012388 \\
-0.021782
\end{bmatrix}
\]

\[
\Delta |V_3| = -0.9203 \times 0.02178 = -0.02
\]

\[
\delta_2^{(2)} = 0.08538 + 0.001914 = 0.08729 \text{ rad. } = 5.00^0
\]

\[
\delta_3^{(2)} = -0.07449 - 0.012388 = -0.08688 \text{ rad. } = -4.98^0
\]

\[
|V_3|^{(2)} = 0.9232 - 0.02 = 0.9032
\]
Thus at the end of second iteration

\[ V_1 = 1.0\angle 0^0 \]
\[ V_2 = 1.1\angle 5.00^0 \]
\[ V_3 = 0.9032\angle -4.98^0 \]

Continuing in this manner the final solution can be obtained as

\[ V_1 = 1.0\angle 0^0 \]
\[ V_2 = 1.1\angle 5^0 \]
\[ V_3 = 0.9\angle -5^0 \]

Once we know the final bus voltages, if necessary, line flows, transmission loss and the slack bus power can be calculated following similar procedure adopted in the case of Gauss–Seidel method.
PROBLEM 1

Consider the transmission network shown. Generators of reactances \( j1.25 \) are connected at busses 3 and 4. The network graph is also shown below.

i) Obtain the bus admittance matrix of the power system.

ii) Neglect the mutual coupling and find the bus admittance matrix.
### Answer

#### $Y_{bus}$

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-j16.75</td>
<td>j11.75</td>
<td>j2.5</td>
<td>j2.5</td>
</tr>
<tr>
<td>2</td>
<td>j11.75</td>
<td>-j19.25</td>
<td>j2.5</td>
<td>j5</td>
</tr>
<tr>
<td>3</td>
<td>j2.5</td>
<td>j2.5</td>
<td>-j5.8</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>j2.5</td>
<td>j5</td>
<td>0</td>
<td>-j8.3</td>
</tr>
</tbody>
</table>

#### $Y_{bus}'$

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-j14.5</td>
<td>j8</td>
<td>j4</td>
<td>j2.5</td>
</tr>
<tr>
<td>2</td>
<td>j8</td>
<td>-j17</td>
<td>j4</td>
<td>j5</td>
</tr>
<tr>
<td>3</td>
<td>j4</td>
<td>j4</td>
<td>-j8.8</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>j2.5</td>
<td>j5</td>
<td>0</td>
<td>-j8.3</td>
</tr>
</tbody>
</table>
**PROBLEM 2** For the transmission network with the following data, determine the bus admittance matrix.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Bus Code</th>
<th>Line Impedance ( z_{km} )</th>
<th>HLCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 - 2</td>
<td>0.02 + j 0.06</td>
<td>j 0.03</td>
</tr>
<tr>
<td>2</td>
<td>1 - 3</td>
<td>0.08 + j 0.24</td>
<td>---</td>
</tr>
<tr>
<td>3</td>
<td>2 – 3</td>
<td>0.06 + j 0.18</td>
<td>j 0.02</td>
</tr>
<tr>
<td>4</td>
<td>2 – 4</td>
<td>0.06 + j 0.18</td>
<td>j 0.02</td>
</tr>
<tr>
<td>5</td>
<td>2 – 5</td>
<td>0.04 + j 0.12</td>
<td>---</td>
</tr>
<tr>
<td>6</td>
<td>3 – 4</td>
<td>0.01 + j 0.025</td>
<td>j 0.015</td>
</tr>
<tr>
<td>7</td>
<td>4 – 5</td>
<td>0.08 + j 0.2</td>
<td>---</td>
</tr>
</tbody>
</table>

\[
\begin{bmatrix}
6.25-j18.72 & -5+j15 & -1.25+j3.75 & 0 & 0 \\
-5+j15 & 10.834-j32.43 & -1.667+j5 & -1.667+j5 & -2.5+j7.5 \\
-1.25+j3.75 & -1.667+j5 & 16.71-j43.198 & -13.793+j34.483 & 0 \\
0 & -1.667+j5 & -13.793+j34.483 & 17.184-j43.758 & -1.724+j4.310 \\
0 & -2.5+j7.5 & 0 & -1.724+j4.310 & 4.224-j11.810
\end{bmatrix}
\]
PROBLEM 3  Obtain the bus admittance matrix of the transmission system with the following data.

**Line data**

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Between buses</th>
<th>Line Impedance</th>
<th>HLCA</th>
<th>Off nominal turns ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 – 4</td>
<td>0.08 + j 0.37</td>
<td>j 0.007</td>
<td>---</td>
</tr>
<tr>
<td>2</td>
<td>1 – 6</td>
<td>0.123 + j 0.518</td>
<td>j 0.010</td>
<td>---</td>
</tr>
<tr>
<td>3</td>
<td>2 – 3</td>
<td>0.723 + j 1.05</td>
<td>0</td>
<td>---</td>
</tr>
<tr>
<td>4</td>
<td>2 – 5</td>
<td>0.282 + j 0.64</td>
<td>0</td>
<td>---</td>
</tr>
<tr>
<td>5</td>
<td>4 – 3</td>
<td>j 0.133</td>
<td>0</td>
<td>0.909</td>
</tr>
<tr>
<td>6</td>
<td>4 – 6</td>
<td>0.097 + j 0.407</td>
<td>j 0.0076</td>
<td>---</td>
</tr>
<tr>
<td>7</td>
<td>6 – 5</td>
<td>j 0.3</td>
<td>0</td>
<td>0.976</td>
</tr>
</tbody>
</table>

**Shunt capacitor data**

<table>
<thead>
<tr>
<th>Bus No. 4</th>
<th>Admittance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>j 0.005</td>
</tr>
</tbody>
</table>
Answer

\[
\begin{pmatrix}
+0.9992 & 0 & 0 & -0.5583 & 0 & -0.4339 \\
-\text{j}4.3925 & 0 & 0 & +\text{j}2.5820 & 0 & +\text{j}1.8275 \\
0 & +1.0214 & -0.4449 & 0 & -0.5765 & 0 \\
-\text{j}1.9546 & +\text{j}0.6461 & 0 & +\text{j}1.3085 & 0 & 0 \\
0 & -0.4449 & +0.4449 & \text{j}8.2715 & 0 & 0 \\
+\text{j}0.6461 & -\text{j}8.1649 & 0 & \text{j}8.2715 & 0 & 0 \\
-0.5583 & 0 & \text{j}8.2715 & +1.1124 & 0 & -0.5541 \\
+\text{j}2.5820 & 0 & \text{j}8.2715 & -\text{j}13.9869 & 0 & +\text{j}2.3249 \\
0 & -0.5765 & 0 & 0 & +0.5765 & \text{j}3.4153 \\
+\text{j}1.3085 & 0 & 0 & 0 & -\text{j}4.6418 & \text{j}3.4153 \\
-0.4339 & 0 & 0 & -0.5541 & 0 & +0.988 \\
+\text{j}1.8275 & 0 & 0 & +\text{j}2.3209 & \text{j}3.4153 & -\text{j}7.6341
\end{pmatrix}
\]
PROBLEM 4

Consider the power system with the following data:

**Line data**

<table>
<thead>
<tr>
<th>Bus code</th>
<th>Line impedance</th>
<th>Bus data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td>0.05 + j0.15</td>
<td></td>
</tr>
<tr>
<td>1–3</td>
<td>0.10 + j0.30</td>
<td></td>
</tr>
<tr>
<td>2–3</td>
<td>0.15 + j0.45</td>
<td></td>
</tr>
<tr>
<td>2–4</td>
<td>0.10 + j0.30</td>
<td></td>
</tr>
<tr>
<td>3–4</td>
<td>0.05 + j0.15</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>PI</th>
<th>QI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>−0.2</td>
</tr>
<tr>
<td>3</td>
<td>−1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>0.3</td>
<td>−0.1</td>
</tr>
</tbody>
</table>

All the buses other than the slack bus are P – Q type. Assuming a flat voltage start and slack bus voltage as $1.04 \angle 0^\circ$, find all the bus voltages at the end of first Gauss–Seidel iteration.

**Answer**

\[
V_1 = 1.04 + j0.0 \\
V_2^{(1)} = 1.01909 + j0.04636 \\
V_3^{(1)} = 1.02802 - j0.08703 \\
V_4^{(1)} = 1.02505 - j0.00923
\]
PROBLEM 5

In the previous problem, let bus 2 be a P – V bus with $|V_2| = 1.04$ p.u.
Compute

$Q_{I_2}, V_2, V_3$ and $V_4$ at the end of first Gauss – Seidel iteration. Assume flat start and $0.2 \leq Q_{I_2} \leq 1.0$ p.u.

Answer

$Q_{I_2} = 0.20799$
$V_2 = 1.05129 + j0.03388$
$V_3 = 1.03387 - j0.08929$
$V_4 = 1.03968 - j0.0149$

PROBLEM 6

Repeat the previous problem taking the limits on reactive powers as

$0.25 \leq Q_{I_2} \leq 1.0$ p.u.

Answer

$Q_{I_2} = 0.25$
$V_2 = 1.05460 + j0.03278$
$V_3 = 1.034473 - j0.08949$
$V_4 = 1.04118 - j0.01540$