# Launch vehicle optimization

**1.Orbital Mechanics for engineering students**

**Chapter – 11: Rocket vehicle dynamics**

**2. Space Flight Dynamics**

**By William E Wiesel**

**Chapter 7 – Rocket Performance**

# Restricted staging in field-free space

No gravity and no aerodynamics

$$\Delta v = I_{sp} g_0 \ln \frac{m_0}{m_f}$$

$$\frac{m_0}{m_f} = e^{\frac{\Delta v}{I_{sp} g_0}}$$

$$\Delta m = m_0 - m_f;$$

$$\frac{\Delta m}{m_0} = 1 - e^{-\frac{\Delta v}{I_{sp} g_0}}$$

Let gross mass of a launch vehicle m0 =

empty mass mE +

propellant mass mp +

payload mass mPL

Empty mass mE = mass of structure + mass of fuel tank and related system + mass of control system

Let us divide the above by m0

We can write as

$$\pi_E + \pi_p + \pi_{PL} = 1$$

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structural fraction , $\pi E = mE / m0$

Propellant fraction, $\pi p = mp / m0$

payload fraction, $\pi PL = mPL / m0$

*Alternately we can define*

Payload ratio $\lambda = \frac{m_{PL}}{m_E + m_p} = \frac{m_{PL}}{m_0 - m_{PL}}$

Structural ratio $\varepsilon = \frac{m_E}{m_E + m_p} = \frac{m_E}{m_0 - m_{PL}}$

Mass ratio $n = \frac{m_0}{m_f}$

Assuming all the propellant Is consumed $n = \frac{m_E + m_p + m_{PL}}{m_E + m_{PL}}$

*λ, ε and n are not independent*

From $\varepsilon = \frac{m_E}{m_E + m_p}$ we can write as

$$m_E = \frac{\varepsilon}{1-\varepsilon} m_p$$

From $\lambda = \frac{m_{PL}}{m_E + m_p}$ we can write as

$$m_{PL} = \lambda(m_E + m_p) = \lambda\left(\frac{\varepsilon}{1-\varepsilon} m_p + m_p\right)$$

$$= \frac{\lambda}{1-\varepsilon} m_p$$

Substituting $$m_E = \frac{\varepsilon}{1-\varepsilon} m_p$$ and

$$m_{PL} = \frac{\lambda}{1-\varepsilon} m_p$$ in

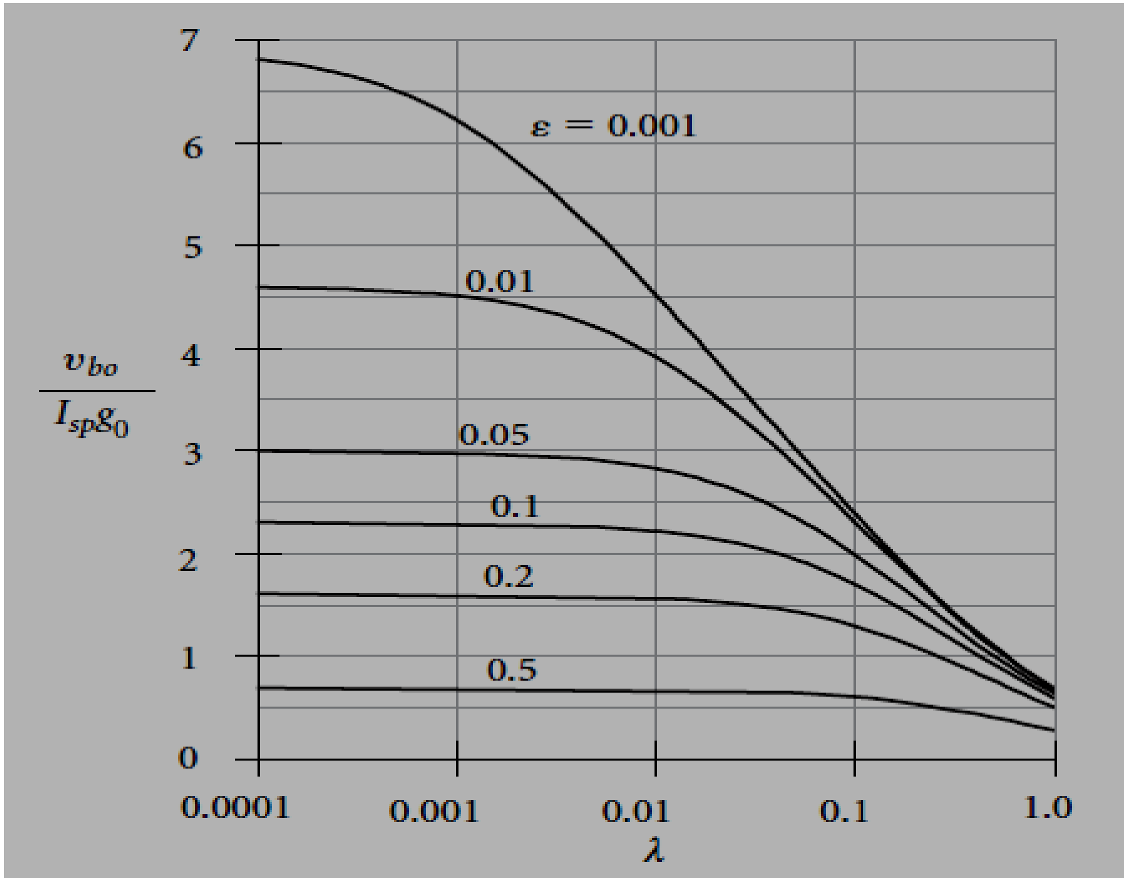
$$n = \frac{m_E + m_p + m_{PL}}{m_E + m_{PL}}$$

We get $$n = \frac{1+\lambda}{\varepsilon+\lambda}$$

Given any two of the ratios *λ, ε and n, we can obtain the third*

Velocity at burn out is

$$v_{bo} = I_{sp} g_0 \ln n = I_{sp} g_0 \ln \frac{1+\lambda}{\varepsilon + \lambda}$$

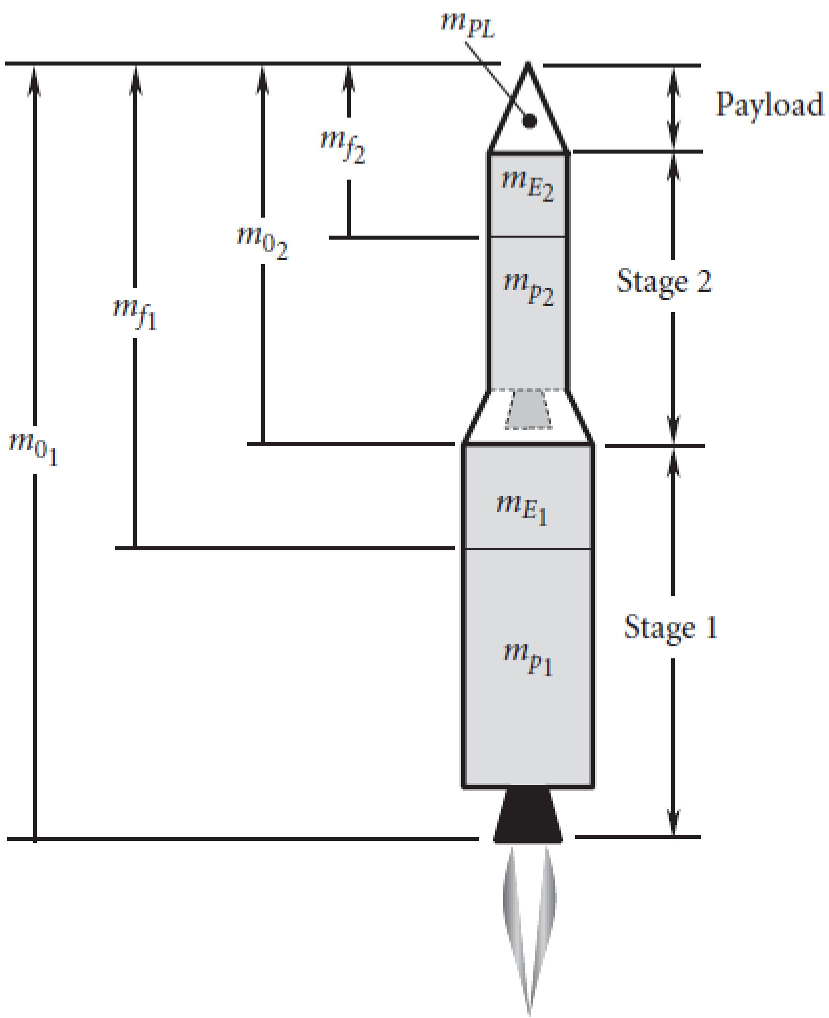


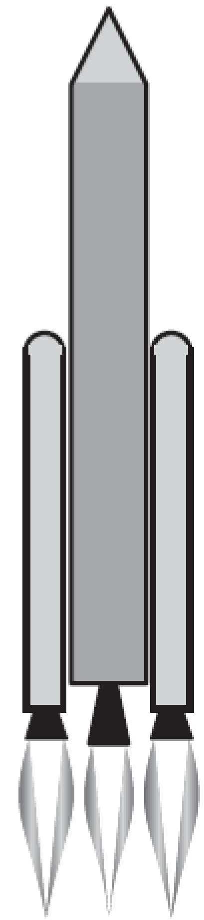
For a given empty mass, the greatest possible $\Delta v$ *occurs when the payload is zero.*

To maximize the amount of payload while keeping the structural weight to a minimum.

Mass of load-bearing structure, rocket motors, pumps, piping, etc., cannot be made arbitrarily small.

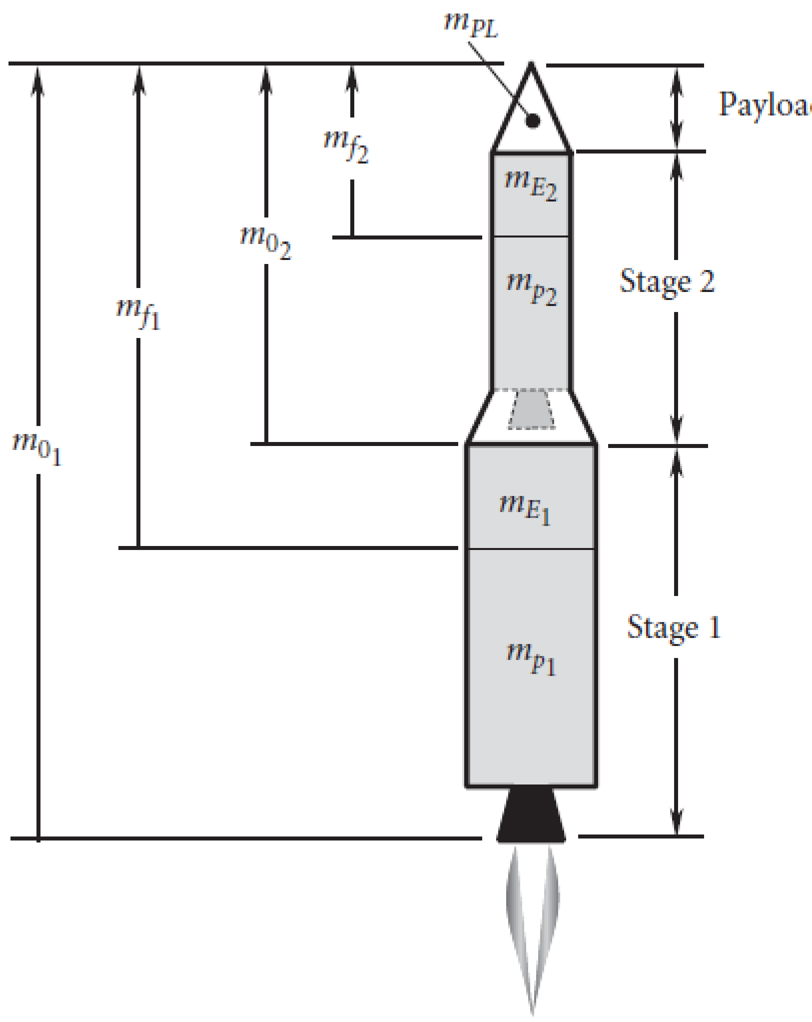
Current materials technology places a lower limit on $\varepsilon$ *of about 0.1.*


$m_{PL}$
Payload
$m_{f_2}$
$m_{E_2}$
$m_{0_2}$
$m_{p_2}$
Stage 2
$m_{f_1}$
$m_{0_1}$
$m_{E_1}$
$m_{p_1}$
Stage 1



# Performance of multistage rocket

Restricted staging - all stages are similar
Each stage has the
same specific impulse Isp
same structural ratio ε
same payload ratio λ.
Hence mass ratios n are identical



Final burnout speed vbo for a given payload mass mPL

Overall payload fraction $\pi_{PL} = \frac{m_{PL}}{m_0}$

*m0 is the total mass of the tandem-stacked vehicle.*

For a single-stage vehicle, the payload ratio is

$$\lambda = \frac{m_{PL}}{m_0 - m_{PL}} = \frac{1}{\frac{m_0}{m_{PL}} - 1} = \frac{\pi_{PL}}{1 - \pi_{PL}}$$

$$\lambda = \frac{\pi_{PL}}{1 - \pi_{PL}}$$

From the equation $$n = \frac{1 + \lambda}{\varepsilon + \lambda}$$

The mass ratio is $$n = \frac{1}{\pi_{PL}(1 - \varepsilon) + \varepsilon}$$

$$v_{bo} = I_{sp} g_0 \ln n = I_{sp} g_0 \ln \frac{1 + \lambda}{\varepsilon + \lambda}$$

$$v_{bo} = I_{sp} g_0 \ln \frac{1}{\pi_{PL}(1 - \varepsilon) + \varepsilon}$$

For a single-stage vehicle burnout speed $v_{bo}$ for a given payload mass $m_{PL}$

$$v_{bo} = I_{sp} g_0 \ln \frac{1}{\pi_{PL}(1-\varepsilon)+\varepsilon}$$

In the book " Space Flight Dynamics" by William E Wiesel
The above expression is given as

$$v_{bo} = -v_e \ln[\varepsilon + (1 - \varepsilon)\pi]$$

Let *m0 be the total mass of the two-stage rocket*

$$m_0 = m_{0_1}$$

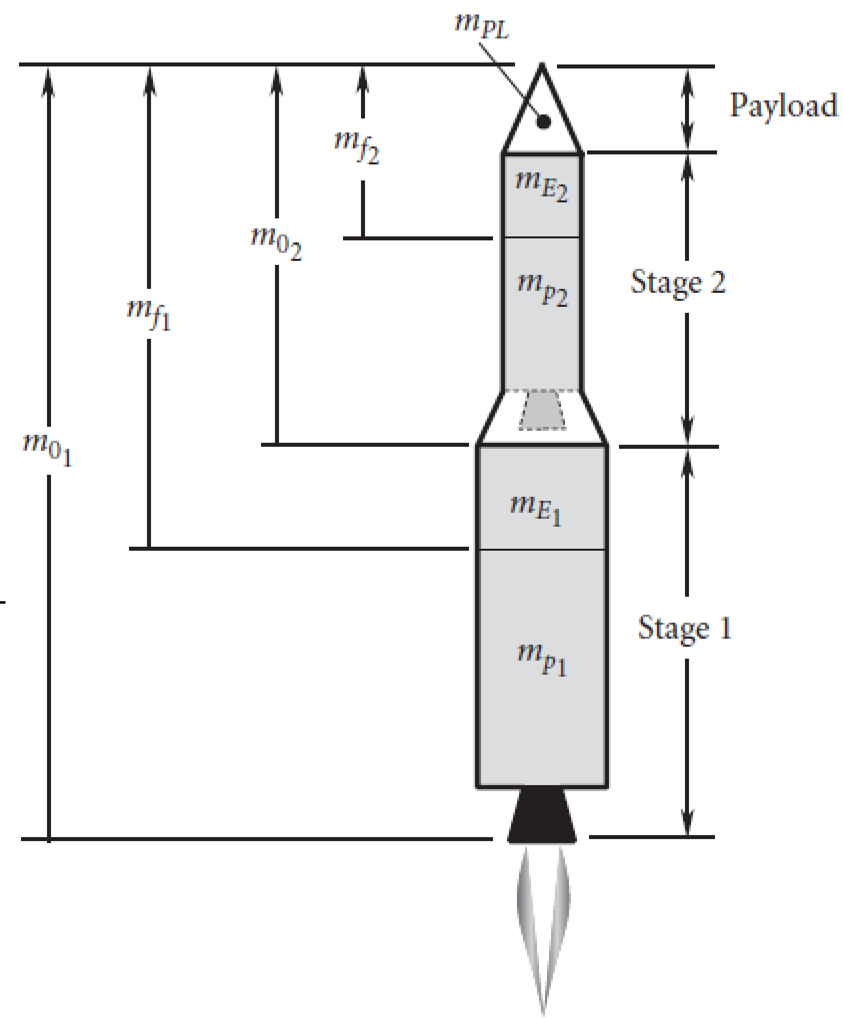
payload of stage 1 is the entire mass *m02 of stage 2*

stage 1 payload ratio is

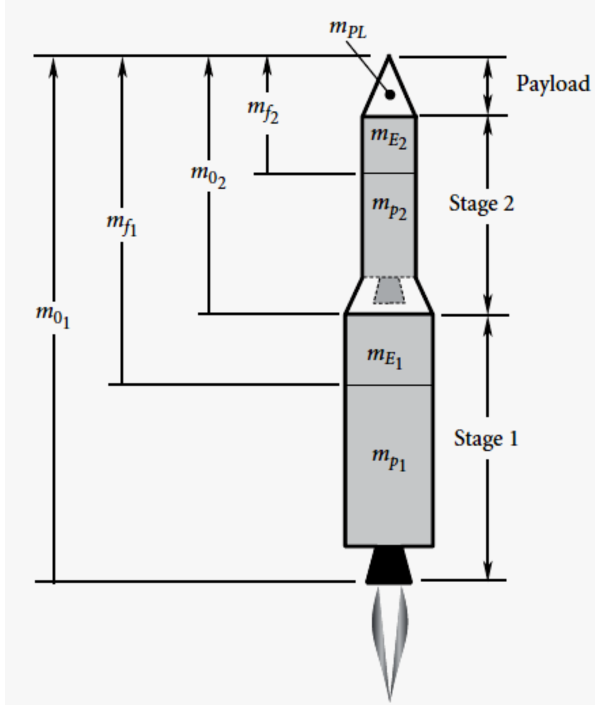
$$\lambda_1 = \frac{m_{0_2}}{m_{0_1} - m_{0_2}} = \frac{m_{0_2}}{m_0 - m_{0_2}}$$

payload ratio of stage 2 is

$$\lambda_2 = \frac{m_{PL}}{m_{0_2} - m_{PL}}$$




And continues………………………




$$m_{E_1} = \frac{\left(1 - \pi_{PL}^{\frac{1}{2}}\right)\varepsilon}{\pi_{PL}} m_{PL} \qquad m_{E_2} = \frac{\left(1 - \pi_{PL}^{\frac{1}{2}}\right)\varepsilon}{\pi_{PL}^{\frac{1}{2}}} m_{PL}$$

$$m_{p_1} = \frac{\left(1 - \pi_{PL}^{\frac{1}{2}}\right)(1-\varepsilon)}{\pi_{PL}} m_{PL} \qquad m_{p_2} = \frac{\left(1 - \pi_{PL}^{\frac{1}{2}}\right)(1-\varepsilon)}{\pi_{PL}^{\frac{1}{2}}} m_{PL}$$

# For a multi stage rockets

$$v_{bo} = \sum_{k=1}^{n} - v_{ek} \ln [\ \varepsilon_k + (1 - \varepsilon_k)\ \pi_k\ ]$$

Calculate the Ve ( go * Isp), $\varepsilon_{k\,,}$ for $\pi_k$ each stage
Sequentially and sum up

Example 11.2 – Page No: 566

The following data is given

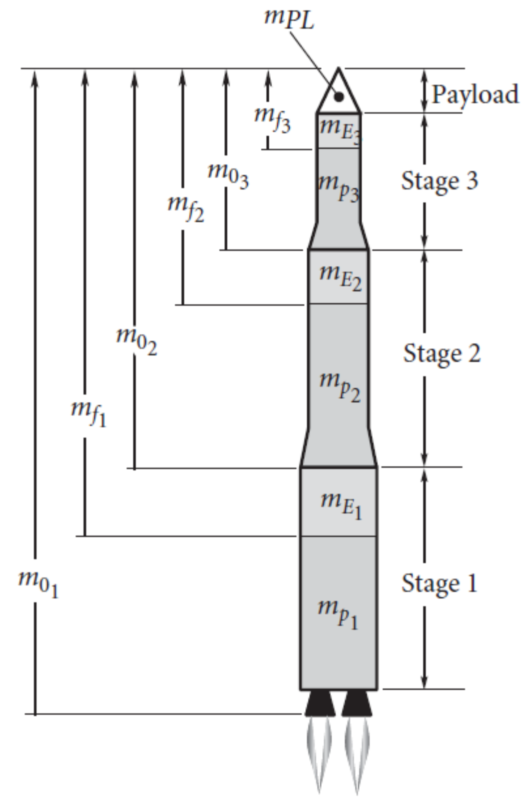
$$m_{PL} = 10\,000\,\text{kg}$$
$$\pi_{\text{PL}} = 0.05$$
$$\varepsilon = 0.15 \quad \text{(a)}$$
$$I_{sp} = 350\,\text{s}$$
$$g_0 = 0.00981\,\text{km/s}^2$$

Calculate the payload velocity $v_{bo}$ at burnout, the empty mass of the launch vehicle and the propellant mass for (a) a single stage and (b) a restricted, two-stage vehicle.

$$m_{p_1} = \frac{\left(1-\pi_{PL}^{\frac{1}{3}}\right)(1-\varepsilon)}{\pi_{PL}} m_{PL}$$

$$m_{p_2} = \frac{\left(1-\pi_{PL}^{\frac{1}{3}}\right)(1-\varepsilon)}{\pi_{PL}^{\frac{2}{3}}} m_{PL}$$

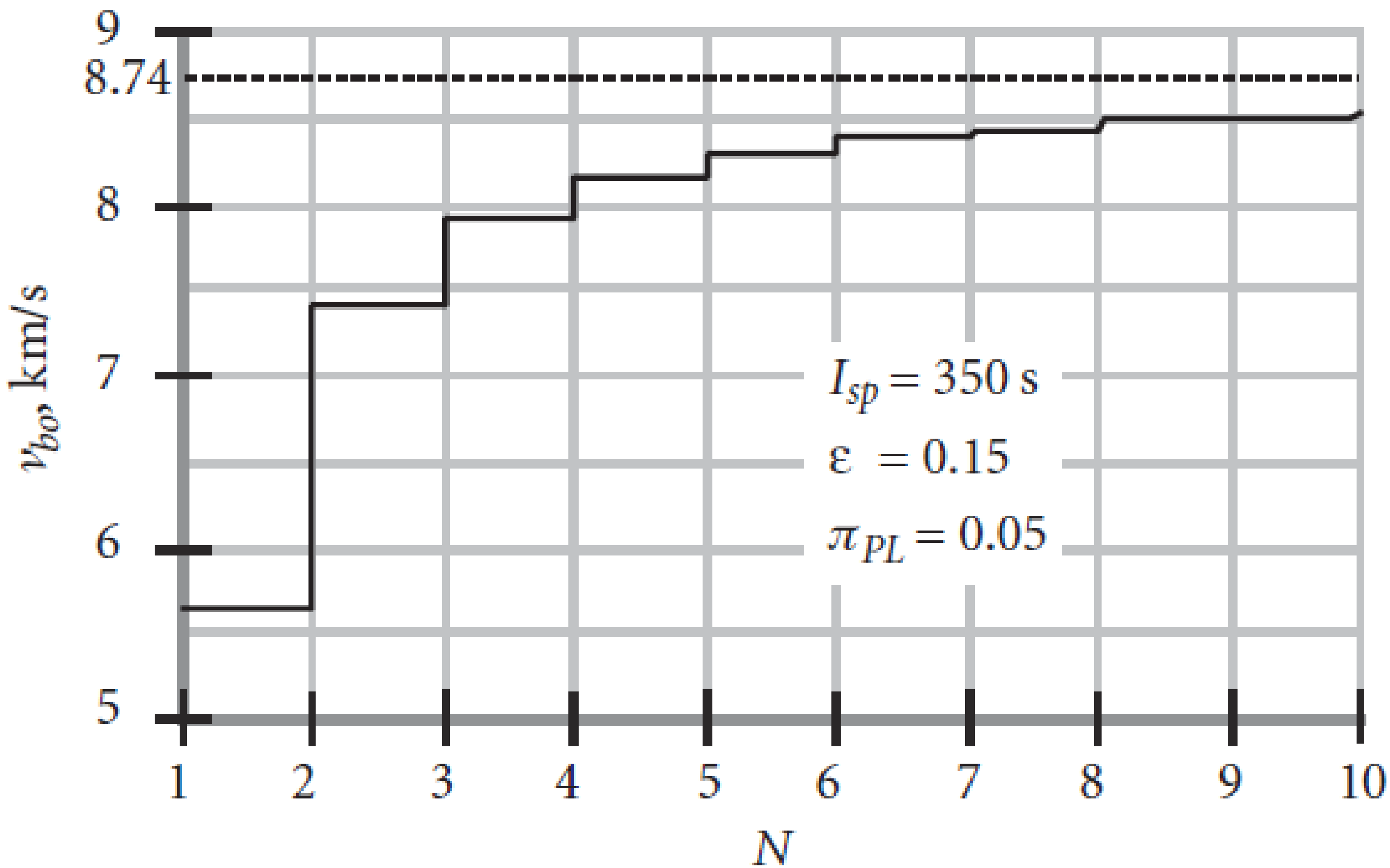
$$m_{p_3} = \frac{\left(1-\pi_{PL}^{\frac{1}{3}}\right)(1-\varepsilon)}{\pi_{PL}^{\frac{1}{3}}} m_{PL}$$




$$m_{E_1} = \frac{\left(1-\pi_{PL}^{\frac{1}{3}}\right)\varepsilon}{\pi_{PL}} m_{PL} \qquad m_{E_2} = \frac{\left(1-\pi_{PL}^{\frac{1}{3}}\right)\varepsilon}{\pi_{PL}^{\frac{2}{3}}} m_{PL} \qquad m_{E_3} = \frac{\left(1-\pi_{PL}^{\frac{1}{3}}\right)\varepsilon}{\pi_{PL}^{\frac{1}{3}}} m_{PL}$$

# Example 11.3

Repeat Example 11.2 for the restricted three-stage launch vehicle.



Zeroth stage – combined I stage
And strapped on boosters

Boosters burn out fast and separated

Balance propellant of the core
I stage burns

How to calculate????????

Assignment!!